A simple laboratory experiment to measure the surface tension of a liquid in contact with air
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Abstract
A simple and accurate laboratory experiment to measure the surface tension of liquids has been developed, which is well suited to teach the behavior of liquids to first- or second-year students of physics, engineering or chemistry. The experimental setup requires relatively inexpensive equipment usually found in physics and chemistry laboratories, since it consists of a used or recycled burette, an analytical balance and a stereoscopic microscope or a micrometer. Experimental data and error analysis show that the surface tension of distilled water, 1-butanol and glycerol can be determined with accuracy better than 1.4%.

Keywords: surface tension, drop weight method, practical sessions

1. INTRODUCTION
Surface tension is a physical parameter that plays an important role in the formation of drops in industrial applications like water atomizers, inkjet printers or fuel injector systems among others. This is due to the fact that surface tension greatly influences the size of the liquid droplets [1]. This parameter is also often measured in the industry for multiple purposes including formulations of antiwrinkle creams, emulsions, foams, lubricants, sprays or inks among others [2]. Diverse types of industries including pharmaceuticals, oil recovery, detergents and soaps, food industry, paper and printing industries among others routinely determine the surface tension of different types of liquid products. It is well known that surface tension depends on diverse variables including the composition of the liquid, the temperature and the surrounding environment [1].

In a liquid in contact with a gas one can distinguish between two types of molecules. The first one corresponds to the inner liquid molecules which are completely surrounded by other liquid molecules. In this case the cohesive forces among the inner molecule and the neighbors are balanced. The second case corresponds to the surface molecules, i.e. the ones located in the boundary between the liquid and the gas. In this case the surface molecule is not completely surrounded by other like neighboring molecules, so it coheres more strongly with those found at the interface between the liquid and the gas, so each surface molecule is attracted by the inner molecules towards the center of the liquid. Therefore, the outer surface of the liquid stays under tension, thus behaving like an elastic thin film [3,4].

Fig. 1 describes this effect related to the cohesive forces between liquid molecules, which results in the surface tension phenomenon [3,5]. These cohesive forces among molecules of the surface layer determine the spherical shape of the droplets. Surface tension has units of force per unit length, N/m or energy per unit area J/m², depending on how surface tension is defined [3] since both of them lead to the same physical units, i.e. N/m = J/m². Liquids in which the molecules interact strongly present higher values of surface tension. It is well known that surface tension usually decreases with increasing temperatures. For example, the cleaning efficiency of water is improved at high temperature because of its lower surface tension, thus becoming a better wetting agent.
Note that the term surface tension is usually applied when there is a single liquid surface, for example in liquid-gas or liquid-solid interface. In the case of two immiscible liquids where there is an interface between the two liquids, this effect is often called interfacial tension.

There are several methods to measure the surface tension, including the drop volume and the drop weight methods [6], the capillary rise method [7], the pendant drop method and the DuNoy ring method among others. According to reference [1], because of its simplicity, accurate results under suitable conditions and small sample size requirements, the drop weight method is amongst the most used methods to measure the surface tension of liquids.

The measurement of surface tension has also attracted the attention of physicists [8]. This physical magnitude can be easily measured using inexpensive and conventional equipment available in most university laboratories achieving a good experimental uncertainty, as shown in this paper. The uncertainty of the experiment proposed in this paper is minimized by applying a correction factor to the original Tate’s law based on experimental data, since it is recognized that when applying the drop weight method only a portion of the pendant drop breaks away from the dripping tip. If this effect is not taken into account, it leads to an underestimation of the surface tension [3]. This paper shows by means of experimental data that the inclusion of this correction factor is very straightforward while improving results accuracy.

The measurement system proposed here is well suited to be applied in undergraduate laboratory sessions or in lecture demonstrations in a wide variety of university courses including physics, chemistry or engineering.

2. THE DROPWEIGHT METHOD

The drop weight method is based on measuring the weight of the drop falling from the bottom of a vertical capillary or tube. It is assumed that the liquid drop suspended from the end of the capillary will fall when the force due to gravity equals the vertical force created by the surface tension. According to references [9,10] Tate’s law is based on the balance between the drop weigh, which pulls the drop down, and the force due to the surface tension, which holds the pendant drop to the tip. Therefore, Tate’s law states that just before detachment and under equilibrium conditions, the weight of the pendant drop equals the vertical component of the force exerted by the surface tension, which is proportional to the wetted perimeter of the boundary between the liquid and the capillary [9,10]

\[ m \cdot g = \gamma \cdot 2 \cdot \pi \cdot r \]  

(1)

\( m \) being the mass of the falling drop, \( g \) the acceleration of gravity, \( \gamma \) the surface tension of the droplet–air interface and \( 2 \cdot \pi \cdot r \) the wetted perimeter. Note that according to the wetting behavior of the analyzed liquid, \( r \) may be either the outer or the inner radius of the capillary.

Therefore, according to equation (1), by measuring the weight of the falling drop the surface tension of a liquid can be obtained experimentally. However, researchers soon realized that Tate’s law lead to deceptively low values of the measured surface tension.

Since detached drops are not always equal, a reasonably large number of drops must be measured in order to calculate the average drop weight [1]. In this work 30 drops are used to calculate the average drop weight, thus ensuring statistical consistence, accuracy and reproducibility in measurements [6]. In addition, the experiment must guarantee that the drop is formed slowly enough to minimize
hydrodynamic effects, which lead to irregular drops at high drop formation rates. According to references [1,3] drop formation times of above 30 s allow avoiding hydrodynamic effects.

Figure 2. Drop forming in the tip of a burette. Forces acting on the drop according to Tate’s law.

Tate’s law supposes an ideal drop, i.e. it assumes that the whole pendant drop is released from the end of the capillary, thus obtaining a falling drop with the same mass than that of the original pendant drop when achieving the maximum stable size. However, experimental results diverge somewhat from this simple analysis since depending on factors including the liquid analyzed or the geometry of the capillarity among others, up to 40% of the drop volume may be left on the capillarity tip [3]. The weight of the detached drop is lower than that predicted by equation (1), so a correction factor \( F \) is usually added to the original Tate’s law,

\[
m \cdot g = F \cdot \gamma \cdot 2 \cdot \pi \cdot r
\]

The most widely accepted correction factor \( F \) is the one published by Earnshaw et al. [11], which was in fact a collection of experimental data recorded by many other authors, including Harkins and Brown [12] and Wilkinson [13] among others. The correction factor \( F \) relates the ratio between the real and the ideal drop volume against the dimensionless capillarity radius \( r/V^{1/3} \), thus resulting in [3],

\[
F(r/V^{1/3}) = V/V_{\text{ideal}}
\]

Experimental values of the correction factor \( F \) may be adjusted with high fidelity in the range 0-1.2 by applying the mathematical fit proposed by Lee–Chan–Pogaku [3] which is used in this paper,

\[
F(r/V^{1/3}) = 1.000 - 0.9121 \cdot (r/V^{1/3})^3 - 2.109 \cdot (r/V^{1/3})^2 + 13.38 \cdot (r/V^{1/3})^3 - 27.29 \cdot (r/V^{1/3})^4 + 27.53 \cdot (r/V^{1/3})^5 - 13.58 \cdot (r/V^{1/3})^6 + 2.593 \cdot (r/V^{1/3})^7
\]

It is worth noting that in the limit, when \( r/V^{1/3} \ll 1 \), Tate’s law is recovered, since equation (4) results in \( F = 1 \).

The variation of the values of the correction factor is minimized when the ratio \( r/V^{1/3} \) is comprised between 0.6 and 1 as shown in Fig. 2. Therefore, to maximize experimental results accuracy it is advantageous to work within this range. It means that for a given liquid, it is recommended to select the capillarity size to match with this interval of values, since the capillarity tip plays a critical role in drop weight devices [6].
3. EXPERIMENTAL SETUP

The procedure to determine the surface tension of a given liquid is as follows:

- Measure the ambient temperature and fill the burette with the liquid to be analyzed.
- The drop formation time is regulated slowly enough to avoid hydrodynamics effects. For the liquids considered in this paper 30 seconds per drop are enough.
- Wait until 30 drops are fully formed and deposited in a vessel. The weight of the vessel and the radius of the burette tip must be known at this stage.
- The vessel containing the 30 drops is weighted with an analytical balance and the average drop weight is calculated.
- The correction factor $F$ is calculated by applying equation (4) once the mean drop volume $V$ is calculated from the mean mass $m$ and the density of the liquid, which can be obtained from the technical literature.
- Finally, the surface tension is calculated by applying equation (2).

To measure the surface tension two recycled and chipped 50 ml burettes have been used. The lower damaged tip of the burettes was cut and polished with P600 sandpaper, ensuring no sharp edges. Burettes were thoroughly cleaned before use.

Table 1 summarized the material required to measure the surface tension of the sample liquids.

<table>
<thead>
<tr>
<th>Table 1. List of material required to measure the surface tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical balance</td>
</tr>
<tr>
<td>Micrometer or x20 stereoscopic microscope</td>
</tr>
<tr>
<td>Used glass burette</td>
</tr>
<tr>
<td>P600 sandpaper</td>
</tr>
<tr>
<td>Sample liquids: distilled water, 1-butanol and glycerol</td>
</tr>
<tr>
<td>Vessel 25ml</td>
</tr>
</tbody>
</table>

Fig. 4 shows the two burettes dealt with. Each burette was later filled with water, 1-butanol and glycerol to measure the surface tension of such liquids. This paper deals with distilled water, 1-butanol and glycerol because they are inexpensive and commercially available liquids, and it is easy to find data in the bibliography of their physicochemical variables at different temperatures.
To determine the surface tension by means of the drop weight method it is required to know the diameter of the dripping tip with high accuracy. Experimental results prove that water, 1-butanol and glycerol drops are formed at the outer perimeter of the dripping tip, so \( r \) is the outer radius of the glass capillarity. Therefore, the outer diameter of the burettes tip was measured by using a x20 Jeulin stereoscopic microscope equipped with a reticule in which one millimeter is divided in 100 parts, thus resulting in a resolution of \( 10^{-5} \) m. Figure 5 shows the photographs of the two burette tips dealt with in this work, which were taken with a 5 megapixels BMS digital camera coupled to the stereoscopic microscope.

After performing the abovementioned measurements, the outer tip radius of burettes A and B were found to be 1.71 and 2.95 mm, respectively. Note that very similar results were obtained when measuring both tip radiuses with a standard micrometer.

4. EXPERIMENTAL RESULTS

This section describes the experimental tests carried out to determine the surface tension of the three analyzed liquids, i.e. distilled water, 1-butanol and glycerol. These tests were performed in a small air-conditioned laboratory at 25 °C.

Since the drops formed when applying the drop weight method are not always identical, to calculate the average drop weight 30 drops were used in each measurement of the three liquids. This procedure also increases the accuracy of this method and ensures reproducibility in measurements. The samples were weighted with a Mettler Toledo AB304-S analytical balance, which has a resolution of 0.1 mg.

As explained in Section 3, two burettes were used to determine the surface tension and three liquids were measured, i.e. distilled water, 1-butanol and glycerol.
Table 2 shows the main data of the drop weight method obtained when using burette A.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Outer radius (mm)</th>
<th>Density (Kg/m³)</th>
<th>Drop mass (kg)</th>
<th>Drop volume (m³)</th>
<th>ργ/ρg</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distilled water</td>
<td>1.71</td>
<td>997.05</td>
<td>5.32 x 10⁻⁵</td>
<td>5.34 x 10⁻⁵</td>
<td>0.45</td>
<td>0.6622</td>
</tr>
<tr>
<td>1-Butanol</td>
<td>1.71</td>
<td>809.50</td>
<td>1.57 x 10⁻⁵</td>
<td>1.95 x 10⁻⁵</td>
<td>0.64</td>
<td>0.6191</td>
</tr>
<tr>
<td>Glycerol</td>
<td>1.71</td>
<td>1258.02</td>
<td>4.55 x 10⁻⁵</td>
<td>3.62 x 10⁻⁵</td>
<td>0.52</td>
<td>0.6472</td>
</tr>
</tbody>
</table>

Once the outer tip radius \( R \), the mean drop mass \( m \), and the correction factor \( F \) are known, the surface tension of the analyzed liquid may be determined by isolating the surface tension \( \gamma \) from equation (2) as,

\[
\gamma = m g / (2 \cdot \pi \cdot r \cdot F)
\]

Finally, from results presented in Table 2 and from (3) the surface tension of the three analyzed liquids is calculated as shown in Table 3. Note that calculations assume \( g = 9.80 \text{ m/s}² \).

<table>
<thead>
<tr>
<th>System</th>
<th>Surface tension ( \times 10^{-3} \text{ N/m} )</th>
<th></th>
<th>Lit. (-) Exp.</th>
<th>Lit. (-) Exp./Lit.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distilled water</td>
<td>72.14 ( ^a )</td>
<td>72.86</td>
<td>0.72</td>
<td>1.00%</td>
</tr>
<tr>
<td>1-Butanol</td>
<td>23.48 ( ^b )</td>
<td>23.20</td>
<td>0.28</td>
<td>1.19%</td>
</tr>
<tr>
<td>Glycerol</td>
<td>64.00 ( ^c )</td>
<td>64.19</td>
<td>0.19</td>
<td>0.29%</td>
</tr>
</tbody>
</table>

Results presented in Table 3 show that the accuracy attained with the applied method is very high, since for all analyzed liquids the error is below 1.2% when compared with literature data.

Table 4 shows the main data of the drop weight method measured when using burette B.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Outer radius (mm)</th>
<th>Density (Kg/m³)</th>
<th>Drop mass (kg)</th>
<th>Drop volume (m³)</th>
<th>ργ/ρg</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distilled water</td>
<td>2.95</td>
<td>997.05</td>
<td>8.29 x 10⁻⁵</td>
<td>8.32 x 10⁻⁵</td>
<td>0.67</td>
<td>0.6118</td>
</tr>
<tr>
<td>1-Butanol</td>
<td>2.95</td>
<td>809.50</td>
<td>2.68 x 10⁻⁵</td>
<td>3.31 x 10⁻⁵</td>
<td>0.91</td>
<td>0.6009</td>
</tr>
<tr>
<td>Glycerol</td>
<td>2.95</td>
<td>1258.02</td>
<td>7.27 x 10⁻⁵</td>
<td>5.78 x 10⁻⁵</td>
<td>0.76</td>
<td>0.6005</td>
</tr>
</tbody>
</table>

Table 5 shows the surface tension results of the three analyzed liquids, which have been attained from experimental data presented in Table 4.

<table>
<thead>
<tr>
<th>System</th>
<th>Surface tension ( \times 10^{-3} \text{ N/m} )</th>
<th></th>
<th>Lit. (-) Exp.</th>
<th>Lit. (-) Exp./Lit.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distilled water</td>
<td>72.14 ( ^a )</td>
<td>71.67</td>
<td>0.47</td>
<td>0.65%</td>
</tr>
<tr>
<td>1-Butanol</td>
<td>23.48 ( ^b )</td>
<td>23.59</td>
<td>0.11</td>
<td>0.45%</td>
</tr>
<tr>
<td>Glycerol</td>
<td>64.00 ( ^c )</td>
<td>63.99</td>
<td>0.01</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

Once again, results presented in Table 5 show a high accuracy in the results attained, since for the three analyzed liquids the error is below 0.6% when compared with literature data.

### 5.1 Experimental uncertainty calculation

The uncertainty of the surface tension as calculated from equation (5) is propagated as,

\[
\Delta \gamma = \frac{\partial \gamma}{\partial m} \Delta m + \frac{\partial \gamma}{\partial g} \Delta g + \frac{\partial \gamma}{\partial r} \Delta r + \frac{\partial \gamma}{\partial F} \Delta F
\]

From equation (6) it results,
\[ \Delta y = \frac{g}{2 \cdot \pi \cdot r \cdot F} \Delta m + \frac{m \cdot g}{2 \cdot \pi \cdot r \cdot F} \Delta g + \frac{m \cdot g}{2 \cdot \pi \cdot r \cdot F^2} \Delta F \]  

(7)

Table 6 shows the different values of the physical magnitudes and their uncertainties as well as the results from equation (7).

<table>
<thead>
<tr>
<th>System</th>
<th>( r ) mm</th>
<th>( \Delta r ) mm</th>
<th>( g ) m/s(^2)</th>
<th>( \Delta g ) m/s(^2)</th>
<th>( m ) x 10(^4) kg</th>
<th>( \Delta m ) x 10(^4) kg</th>
<th>( F ) -</th>
<th>( \Delta F ) mN/m</th>
<th>( \gamma ) mN/m</th>
<th>( \Delta \gamma ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.71</td>
<td>0.01</td>
<td>9.80</td>
<td>0.01</td>
<td>5.3</td>
<td>0.1</td>
<td>0.6662</td>
<td>3</td>
<td>72.86</td>
<td>0.67</td>
</tr>
<tr>
<td>Water</td>
<td>2.95</td>
<td>0.01</td>
<td>9.80</td>
<td>0.01</td>
<td>8.3</td>
<td>0.1</td>
<td>0.6118</td>
<td>3</td>
<td>71.67</td>
<td>0.44</td>
</tr>
<tr>
<td>1-Butanol</td>
<td>1.71</td>
<td>0.01</td>
<td>9.80</td>
<td>0.01</td>
<td>1.6</td>
<td>0.1</td>
<td>0.6191</td>
<td>3</td>
<td>23.20</td>
<td>0.32</td>
</tr>
<tr>
<td>1-Butanol</td>
<td>2.95</td>
<td>0.01</td>
<td>9.80</td>
<td>0.01</td>
<td>2.7</td>
<td>0.1</td>
<td>0.6099</td>
<td>3</td>
<td>23.59</td>
<td>0.20</td>
</tr>
<tr>
<td>Glycerol</td>
<td>1.71</td>
<td>0.01</td>
<td>9.80</td>
<td>0.01</td>
<td>4.6</td>
<td>0.1</td>
<td>0.6472</td>
<td>3</td>
<td>64.19</td>
<td>0.61</td>
</tr>
<tr>
<td>Glycerol</td>
<td>2.95</td>
<td>0.01</td>
<td>9.80</td>
<td>0.01</td>
<td>7.3</td>
<td>0.1</td>
<td>0.6005</td>
<td>3</td>
<td>63.99</td>
<td>0.40</td>
</tr>
</tbody>
</table>

a Reticule uncertainty
b From [18]
c Analytical balance uncertainty
d From [11]

Note that results shown in Table 6 agree well with those presented in Tables 3 and 5.

5. CONCLUSIONS

This work has shown that with very simple and relatively inexpensive equipment usually found in physics and chemistry laboratories, it is possible to measure the surface tension of liquids with an accuracy better than 1.4% for all samples analyzed. The procedure presented fits well to develop practical sessions in undergraduate lecture demonstrations or laboratory practicals in different university courses including physics, chemistry or engineering among others. Furthermore, due to the good results achieved, it is also possible to apply this system for research work.

REFERENCES

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