



# Modeling inertia through the interaction with quantum fluctuations

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## ABSTRACT

The origin of inertia of macroscopic bodies has never been thoroughly elucidated. In this paper we provide a new explanation based on the following assumptions: (i) we can think of any body as being composed by resonant parts of Planck size, (ii) inertia arises from the interaction among these elementary constituents and quantum fluctuations. In compliance with such prescription, we propose two frameworks within which inertia can be modeled. The first one relies on the direct application of Heisenberg Uncertainty Principle to the fluctuations nearby a body, the other involves the asymmetric (Casimir-like) damping of the radiation experienced by an accelerated object due to the appearance of a Rindler horizon. Consistency between the two approaches is then discussed.

## Introduction

*Inertia* is the tendency of physical objects to resist any change in their state of motion. More practically, we can say that it is the property that lets objects stay still if they are still, or keeps them moving if they are moving. From Aristotle's general considerations on natural motion [1] to Galileo's experiments on falling objects and inclined planes, the understanding of the very nature of inertia has always attracted extensive attention, culminated with the formulation of Newton's laws of dynamics. Nevertheless, in spite of formalizing the definition of inertia, such laws did not tackle the issue of its physical origin.

Two hundred years after the development of Newton's theory, it was Mach who seriously approached the matter. Criticizing Newton's concepts of absolute space and time, he proposed that the inertia of bodies was holistically caused by their interactions with the rest of the Universe. Although Mach's principle (as later loosely termed by Einstein) was soon set aside due to inconsistencies with the then emerging relativity theory, its impact was so much influential that a number of physicists tried to investigate the origin of inertia in greater detail. Amongst them, we mention the proposal by Sciama [2] and Dicke [3], who ascribed inertia to a field contact inductive effect of distant matter, and the attempt by Moon and Spencer [4] (later improved by Brown [5]), who introduced the concept of retarded action-at-a-distance of cosmic matter on objects in the laboratory (see Ref. [6] for a detailed review on the topic).

From classical shores, in recent years the debate on the origin of inertia has landed on more quantum grounds. For instance, in Ref. [7]

Haisch et al. suggested a model for inertia that uses the electromagnetic part of Unruh radiation, which is the radiation perceived by a uniformly accelerated probe in the inertial vacuum [8]. The idea is that, due to the interaction with the zero-point field, oscillating partons within an accelerated object feel a magnetic Lorentz force which opposes the acceleration, behaving just like inertia. In a similar vein, inspired by previous works by Milgrom [9], McCulloch conjectured a modification of the inertial mass resulting from Unruh radiation imbalance between the cosmic and Rindler horizons (Quantized Inertia) [10]. However, both these two approaches have been partly criticized [11,12], thus leaving the problem of the origin of inertia still open.

Starting from the outlined picture, in this work we propose a new model for inertia based on the following assumptions: (i) we suppose that any body can be conceived as a collection of resonant parts of Planck size (this is in line with the hypothesis that Planck scale has a fundamental rôle in the quantization of space-time [13] and we assume that the same happens for the energy and the mass of macroscopic bodies and their interactions in a quantification at a deeper level), (ii) inertia is the result of the interaction of these components with vacuum fluctuations (since at Planck level these fluctuations become relevant and, in the same way as for the Casimir force, they produce macroscopic effects). The first assumption generalizes the corpuscular picture of black holes [14,15] to any macroscopic body [16]. The second requirement is the core of our analysis, as it provides us with a recipe to trace the origin of a macroscopic property like inertia back to a more fundamental microscopic mechanism. Within this framework,

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we present two derivations of inertia which turn out to be two sides of the same coin. The first one is investigated in Section “Inertia from Heisenberg Uncertainty Principle” and involves the application of Heisenberg Uncertainty Principle (HUP) and the particle event horizon description of macroscopic bodies. On the other hand, in Section “Inertia from Rindler-scale Casimir effect” we revisit McCulloch’s theory of QI, showing that inertia can be modeled by the interaction of accelerated bodies with the asymmetrically damped radiation they perceive due to the appearance of Rindler horizon (Rindler-scale Casimir effect). Consistency between the two approaches is then discussed. Conclusions and outlook are summarized in Section “Conclusions and Outlook”. Throughout all the paper, we shall denote by  $\ell_p = \sqrt{\hbar G/c^3}$  and  $m_p = \sqrt{\hbar c/G}$  the Planck length and mass, respectively ( $\hbar$ ,  $c$  and  $G$  are the (reduced) Planck’s constant, the speed of light in vacuum and the gravitational constant).

### Inertia from Heisenberg Uncertainty Principle

We assume that the inertial mass of bodies is given by quantum fluctuations spontaneously popping out around it. It is well-known that these fluctuations can be regarded as the temporary appearance of virtual particle–antiparticle pairs, which annihilate shortly after their creation in a time interval  $\Delta t$ . Denoting by  $\Delta E$  the energy of each fluctuation, from the HUP it follows that

$$\Delta x \Delta p \simeq \frac{\hbar}{2} \implies \Delta E \simeq \frac{\hbar c}{2\Delta x}, \tag{1}$$

where we have used  $\Delta p \simeq \Delta E/c$  and  $\Delta x \simeq c\Delta t$  is the distance over which the fluctuation is allowed to propagate. Henceforth, we shall denote this distance by  $\Delta x = r$ . Clearly, the higher the energy, the shorter the distance traveled by the fluctuation, and vice-versa. This means that the only virtual particles which manage to reach the body are those originating in its neighborhood or of very low energy. To account for this intrinsic cutoff, we introduce an effective radius  $r_e$  [17], representing the threshold beyond which quantum fluctuations can be safely neglected.

Following Ref. [16] and therein, let us now depict a generic macroscopic body as a collection of elementary resonant parts of Planck size  $\ell_p = \sqrt{\hbar G/c^3}$ , which acts as minimal length scale in our quantum picture of the gravitational interaction. The specific features of the interaction of quantum fluctuations with these resonant elements are still object of investigation. Here we just give a simple model in a completely inelastic collision that must be confirmed by further works in this direction. The most relevant result is that the two approaches to inertia we consider (the one given in this section and the other in the next section) lead to very similar outcomes.

From Eq. (1), the force exerted by a vacuum fluctuation on each resonant part is then

$$F(r) = -\frac{\partial(\Delta E)}{\partial r} \simeq \frac{\hbar c}{2r^2}. \tag{2}$$

corresponding to an amount of transferred energy on each resonant part equal to

$$E = \int_{\ell_p}^{r_e} F(r) dr \simeq \frac{\hbar c}{2} \left( \frac{1}{\ell_p} - \frac{1}{r_e} \right) \simeq \frac{\hbar c}{2\ell_p}. \tag{3}$$

In the second step we have exploited the fact that the Planck length is expected to be much smaller than the effective radius  $r_e$ , i.e.,  $\ell_p \ll r_e$ , and that the closer the fluctuation, the higher it contributes to  $E$ .

At this stage, it is worth introducing the so-called *particle event horizon* description of elementary particles. According to this scheme, since elementary particles represent field singularities, they can be interpreted as black holes. Then, by the scale invariance of the general relativity, it is possible to formulate an Einstein-like equation which allows us to derive all their fundamental properties (see Ref. [18] and therein). Consequently, one finds that any elementary particle is confined within a region of binding surface  $A = 4\pi r_s^2$ , where  $r_s =$

$2GM/c^2$  is the Schwarzschild radius coming from the singularity of the exact solution of Einstein’s equation for the gravitational field outside of a non-rotating, spherically symmetric body of mass  $M$ . As explained in Ref. [16], this model can be extended to macroscopic bodies as well. Naively speaking, in this case one can associate to any object a event horizon with a radius given by the Schwarzschild radius of the equivalent black hole having the same mass.

With the above scheme in mind, let us consider a macroscopic body of mass  $M$ . It is a simple matter to understand that the number of its resonant parts of Planck size is then given by the ratio of the associated Schwarzschild radius to the Planck length, i.e.,

$$n \simeq \frac{r_s}{\ell_p} = \frac{2M}{m_p}, \tag{4}$$

which can be inverted to give

$$M \simeq n \frac{m_p}{2}. \tag{5}$$

Here we have denoted by  $m_p = \sqrt{\hbar c/G} = \hbar/(c\ell_p)$  the Planck mass. Notice that, in the framework of Corpuscular Gravity (CG) theory [14], black holes are described as Bose–Einstein condensates of  $N$  gravitons stuck at the critical point. In light of the above correspondence between generic macroscopic bodies and black holes, we can then rephrase also the concept of resonant parts in the language of CG. Following Ref. [14], we would have

$$n = \sqrt{N}. \tag{6}$$

It is now easy to prove that Eq. (5) coincides with the expression of inertia arising from the vacuum energy (3) which arrives on the body for each elementary component. Indeed, by assuming that the macroscopic body has  $n$  resonant parts and that it owes its inertia to the interaction with vacuum fluctuations, the mass equivalent to the energy  $E$  that can interact with the environment is

$$n \frac{E}{c^2} \simeq n \frac{m_p}{2} = M. \tag{7}$$

Therefore, in agreement with the CG picture [14], the inertial mass of a body can be conceived as the result of the interaction among the gravitational fluctuations popping out around the body and its resonant parts of Planck size (or gravitons in a Bose–Einstein condensate). In a broader sense, this is a realization of the old Mach’s principle, already sought after by Einstein, which stated that the inertia of a body could be somehow influenced by the rest of the Universe (more precisely, Mach referred to the background of distant stars that allow to fix the inertial reference systems). This conjecture did not make sense anymore (and was abandoned by Einstein himself) after the development of relativity theory. However, its statement is now realized but with the quantum fluctuations that actually give the fundamental contribution to the energy of the Universe. Notice that a similar attempt to establish a connection between inertia and vacuum fluctuations has been performed in Quantum Field Theory in Ref. [19] by revisiting the relationship between the mass of charged particles and zero-point electromagnetic fields.

Before proceeding with the next modeling of inertia, let us come back to the concept of resonant parts of Planck size. For a given body, one may wonder how to relate this microscopic property to more familiar macroscopic features, independently of Eq. (4). In this regard, the calculation of the entropy comes to our aid. In statistical mechanics, it is well-known that entropy is a measure of the number of possible microstates corresponding to the system’s macrostate. As shown in Ref. [14], for the particular case of CG black holes one has

$$S_{\text{bh}} = \log \xi_{\text{bh}}^N \simeq N = n^2, \tag{8}$$

where  $\xi_{\text{bh}}$  is the number of possible states for each of the  $N$  gravitons in the condensate (we have set Boltzmann’s constant equal to one) and we have used Eq. (6). Similarly, for a generic macroscopic body of the same mass, we can write

$$S = \log \xi^N = N \log \xi, \tag{9}$$

where now  $\xi$  is the number of states for each graviton in the body. The problem is how to quantify this number, since we know only that  $\xi \gg \xi_{bh}$ . To infer an estimate for  $\xi$ , let us then follow this reasoning: suppose to consider two systems, the one made of a single atom of an heavy element, for example lead with atomic number  $Z = 82$ , the other made of atoms of a lighter element, for example 82 atoms of hydrogen with  $Z = 1$ . Although the two systems have the same number of excitable (resonant) states, *i.e.* 82, due to Pauli exclusion principle the total number of states in which the electrons can be arranged is clearly higher for the system made of hydrogen atoms. In other terms, we expect that the lower the density, the higher the total number of accessible states for the system and vice versa. Let us then assume  $\xi \simeq \kappa/\rho$ , where  $\rho$  is the density of the body and  $\kappa$  a suitable constant. By plugging into Eq. (9), we get

$$S = N \log(\kappa/\rho). \tag{10}$$

Notice that the constant  $\kappa$  can be fixed by requiring that the above relation reduces to Eq. (8) for  $\rho = \rho_{bh}$ , which implies  $\kappa = \rho_{bh} \xi_{bh}$ . In this way, we obtain

$$\begin{aligned} S &= N \log(\rho_{bh} \xi_{bh}/\rho) \simeq N [1 + \log(\rho_{bh}/\rho)] \\ &= n^2 [1 + \log(\rho_{bh}/\rho)]. \end{aligned} \tag{11}$$

This provides us with the relation we were looking for, since it links the number of resonant parts of a body conceived as a condensate of gravitons to its entropy and density.

### Inertia from Rindler-scale Casimir effect

Let us consider a body of mass  $M$  moving rightwards along the  $x$ -axis with uniform (proper) acceleration  $\mathbf{a} = a\hat{i}$  (see Fig. 1).<sup>1</sup> As a result of this motion, it is well-known that a dynamic (Rindler) horizon appears at a distance  $c^2/a$  on the left side of the body, since information coming from farther away can never catch up with the body (Rindler-scale Casimir effect). Hence, the impact of vacuum fluctuations turns out to be weaker from the left than the right side (where no shielding occurs), giving rise to a net force which pushes the object back against its acceleration. Notice that in the original framework of Quantized Inertia (QI) [20], McCulloch ascribes inertia to the damping of fluctuations of Unruh radiation at temperature [8]

$$T_U = \frac{\hbar a}{2\pi c}, \tag{12}$$

due to both Rindler and cosmic horizons, the latter appearing at a distance far greater than the former and being relevant only on the right side of the body. In that case, the author obtains a modified expression for the inertial mass, with a correction scaling as  $2c^2/(a\theta)$ , where  $\theta \simeq 10^{26}$  m is the Hubble diameter. Nevertheless, since in the present study we are interested in explaining the origin of pure inertia, we shall consider accelerations large enough to neglect this Hubble-scale Casimir effect and the ensuing correction to inertia.<sup>2</sup> This amounts to saying that  $\theta$  is much larger than any other characteristic length scale in our analysis.

Let us show how this asymmetric Rindler-scale Casimir effect can model inertia intuitively. In this regard, we recall that, in the case of an isotropic radiation, the pressure exerted on the each resonant part of Planck size of the body would simply be

$$F = \frac{uA}{3}, \tag{13}$$

<sup>1</sup> We remark that, due to the accelerated expansion of the Universe, any body has an acceleration at least given by the cosmic acceleration.

<sup>2</sup> Notice that  $2c^2/\theta \simeq 10^{-10}$  m/s<sup>2</sup>, which means that for accelerations of the order of Earth's gravity, we are by far in the regime where corrections to the inertia can be neglected.

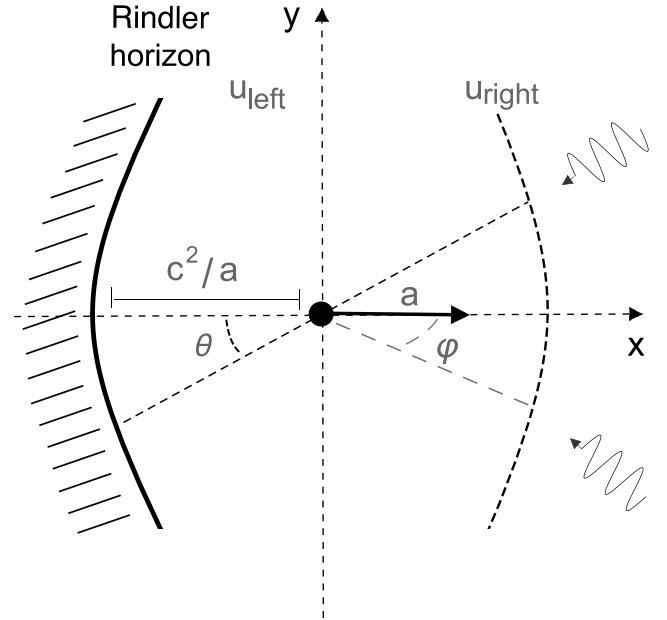


Fig. 1. Schematic representation of the origin of inertia: a body (black dot) moving rightwards with acceleration  $a$  experiences a Rindler horizon far away to its left, at a distance  $c^2/a$  (the solid curve). The net contribution to the inertia is given by those fluctuations which originate at a distance greater than  $c^2/a$  to its right (beyond the dashed line). The radiation pressure imbalance will produce a force against the direction of acceleration.  $\theta$  is the angle of integration in the  $(x, y)$ -plane,  $\varphi$  the azimuthal angle.

where  $u$  denotes the radiation energy density and  $A$  is the surface area of the resonant part intercepted by the radiation. However, in our case we need to estimate the net difference between the forces acting on the body from the left and the right, respectively. This can be done by considering a virtual line through the particle forming an arbitrary angle  $\theta$  with the  $x$ -axis (Fig. 1) [20,21]. From Eq. (13), the calculation of the infinitesimal contribution to the net force yields

$$\begin{aligned} dF &= \frac{(u_{left} - u_{right}) A}{3} \\ \implies dF_x &= \frac{(u_{left} - u_{right}) A \cos \theta}{3}, \end{aligned} \tag{14}$$

where in the last step we have taken the component of the force along the  $x$ -axis. Here we have denoted by  $u_{left(right)}$  the radiation energy density in the left (right) region. Clearly, the relative sign arises from the fact that while on the left side the radiation pressure pushes the body from the rear, in the other region it opposes the acceleration.

Now, since the radiation coming from the right does not experience any damping, its energy density is simply given by  $u_{right} = u$ . On the other side, the presence of Rindler horizon reduces the radiation that impact onto the body through the mechanism explained above. This effect has been quantified in Refs. [10,20] and partially corrected in Ref. [21], showing that

$$u_{left} = u \left( 1 - \lambda_U^{peak} \frac{a \cos \theta}{8c^2} \right), \tag{15}$$

where  $\lambda_U^{peak}$  is the peak wavelength of Unruh spectrum. Notice that in Ref. [12] it has been argued that the above formula is incorrect for accelerations around  $a_p \simeq 10^{-9}$  m/s<sup>2</sup>. This has been shown by deriving Planck's law in a cavity and numerically computing the ratio of the discrete to continuous blackbody radiances. However, for  $a \ll a_p$  and  $a \gg a_p$  (that is the regime under investigation in this work), the result of Ref. [12] has approximately the same behavior as the one in Eq. (15), which justifies the use of this relation in our calculations.

By plugging into Eq. (14), after simple algebra we have

$$dF_x = -\frac{u\lambda_U^{peak} A \cos^2 \theta}{24c^2} a, \tag{16}$$

where the minus sign indicates that the net force always opposes the acceleration, as stated above.

In order to derive the total force, we have to add up the contributions from all angles. This first requires an integration over the azimuthal angle  $\varphi$  from  $-\theta$  to  $\theta$ , which trivially gives a factor  $2\theta$ . The result must then be integrated over  $\theta$  from 0 to  $\pi/2$  and doubled in order to span all the  $(x, y)$ -plane. We obtain

$$F_x = -\frac{(\pi^2 - 4) u\lambda_U^{peak} A}{96 c^2} a. \tag{17}$$

We remark that the above calculation differs from the one in Refs. [20, 21], where the integration over the azimuthal angle  $\varphi$  is performed over the range  $[0, \pi]$ , resulting in a incorrect numerical factor.

Let us now focus on the estimation of the energy density  $u$ . The classical energy density is defined as the energy stored in a given region of space per unit volume. However this definition must be modified in its quantum version, since the volume under consideration cannot be taken arbitrarily small. This is because the energy comes from quantum fluctuations and these cannot occur at any point in a continuous space. If this were the case, the contribution of any volume would then be an infinity of the same order (it is the same as the number of real numbers in any interval of the real line). To resolve the contradiction, it must be required that there exists a minimum length and therefore a minimum volume. In turn, the space-time discretization poses another problem, since it usually breaks Lorentz invariance. However, the Causal Set theory discretizes space-time without breaking Lorentz symmetry. This is due to the fact that Causal Set theory discretizes the causal structure of space-time using the ideas of Hawking, Malament and Sorkin. In this framework, each element of the causal set has a Planck volume, see for instance Ref. [13] and therein. Therefore, it emerges that the space-time should exhibit a continuous-to-discrete transition at very fine scales, the effective threshold being represented by the Planck scale. This is also pointed out by most of candidate theories of quantum gravity, see for instance Ref. [22]. As a result, we have that the correct evaluation of the density energy in the quantum realm must involve the computation of the energy in a Planck volume, but inside this minimum volume only a single quantum fluctuation can happen. Now we analyze what is the wavelength  $\lambda_p$  of each particle of this fluctuation.

Here we closely follow the arguments of Ref. [23]. As stated above, the fluctuations which are actually responsible for the radiation imbalance are those appearing beyond the hypothetical Rindler horizon in the right region (see Fig. 1). Clearly, the uncertainty in the position of a photon from this region is  $\Delta x \geq \pi c^2/a$ , assuming a form of half sphere for Rindler horizon. Then, very straightforward considerations allow one to derive the following condition on the temperature  $T = \Delta E$  of these fluctuations [23]

$$T \leq T_U, \tag{18}$$

where the Unruh temperature  $T_U$  corresponds to the temperature of those fluctuations coming from Rindler horizon at a distance  $c^2/a$  from the body. Accordingly, denoting by  $E_p = \Delta E/2$  the energy of each photon of any virtual pair, we have  $E_p \leq E_U$ , with  $E_U = T_U/2$  being the energy of a photon of Unruh radiation. But, since  $E_p = hc/\lambda_p$ , it follows that  $\lambda_p \geq \lambda_U$  and it is also true that  $\lambda_p^{peak} \geq \lambda_U^{peak}$ , where  $\lambda_p^{peak}$  is the peak wavelength of the spectrum of fluctuations in all elementary volumes of Planck size. So, we can assume that  $\lambda_p^{peak} = k\lambda_U^{peak}$ , where  $k \geq 1$ .

According to the previous discussion, the energy density can now be written as

$$u = \frac{E}{V} \simeq \frac{hc}{k\lambda_U^{peak} V}, \tag{19}$$

where  $V$  is the volume of the elementary block of the discrete space, which is assumed to be spherical, i.e.  $V \simeq 4/3\pi\ell_p^3$ . In this way, Eq. (17) becomes

$$F_x \simeq -\frac{(\pi^2 - 4)\pi m_p}{32k} a \simeq -\frac{0.6m_p}{k} a, \tag{20}$$

where we have approximated the generic resonant part of the body to a sphere of radius equal to the Planck length  $\ell_p$ , so that the area  $A$  intercepted by the radiation imbalance is simply  $A \simeq 2\pi\ell_p^2$  (it should be reminded that the net radiation only impinge on the right side of the body). From the above equation, it follows that the mass of each resonant part due to the radiation imbalance is nothing but  $0.6m_p/k$ , leading to the total inertial mass

$$M \simeq n \frac{0.6m_p}{k}. \tag{21}$$

It is worth emphasizing that this expression coincides with Eq. (5), up to a numerical factor  $k$ . We can pick its exact value by requiring consistency between Eqs. (7) and (21), obtaining  $k \simeq 1.2 \gtrsim 1$ . In turn, this implies that the wavelength of the fluctuation spectrum which mainly contributes to the energy density  $u$  is  $\lambda_p^{peak} \simeq 1.2\lambda_U^{peak} > \lambda_U^{peak}$ , consistently with our previous considerations. Concerning the comparison between the two models of inertia, we also remark that in the first approach we only take into account all the quantum fluctuations around the body that can interact simultaneously with it (the restriction is given by the number of resonant parts). We can perform this analysis by considering either the body at rest or moving with a certain acceleration. In the first case, the forces in any two opposed directions are equal to each other and the body is kept at rest. On the other hand, if the body is initially accelerated, the situation would be similar to that depicted in the second model, i.e. there would be an unbalance in the radiation produced by the quantum fluctuations that oppose the movement. This happens because the radiation coming from the direction opposed to the movement has to travel longer distances and consequently is less energetic (see the considerations below Eq. (1)). By contrast, in the direction of the movement there are high-energy fluctuations that could not reach the body at rest, but now they can. Hence, a force opposed to the movement would appear, similarly to what discussed for the asymmetric Casimir effect.

Some comments are in order here: first, we notice that the computation of the energy density carried out in Refs. [10,20,21] is not properly justified, since the volume  $V$  that appears in Eq. (19) is not the volume of the particle (as instead assumed in [10,20,21]), but rather it is the volume of the space with respect to which the energy density is estimated. On the other hand, the considerations of Ref. [12] do not take into account the crucial feature of space-time discretization, thus leading to the conclusion that the peak wavelength contribution tends to zero for large accelerations. Second, we stress that in our model inertia is ascribed to the asymmetric pressure of fluctuations appearing beyond Rindler horizon, rather than Unruh radiation on its own. The latter is indeed isotropic and extremely faint, as it only consists of those fluctuations originating very close to the horizon. By contrast, the radiation imbalance involves fluctuations coming anisotropically from an infinite volume (the region beyond the dashed line in the right side of Fig. 1). Thus, just like the Casimir effect, it is perfectly eligible to be at the root of a macroscopic phenomenon like inertia.<sup>3</sup> In this context, we would like to emphasize a meaningful comparison

<sup>3</sup> One might wonder why the standard Casimir effect between two plates is typically so weak, while the Rindler-scale Casimir effect gives rise to an easily observable phenomenon like inertia. In our model, this can be explained by noticing that in the first case the resonant parts of Planck size of the plates do not play any rôle in determining the difference between the inward and outward radiation pressure responsible for the Casimir attraction. For instance it does not matter the mass or the density of the plates. In other terms, each plate acts as a unique resonant part of a certain surface pushed by the boiling vacuum energy. On the other hand, in the Rindler-scale Casimir effect we

between the quantum radiation imbalance and the classical *inertial forces*. Concerning these forces, it is well-known that, in spite of being experienced only by accelerated observers, their effects can be somehow “deduced” by external (*i.e.* inertial) observers as well. In fact, it is a everyday experience that, when a car accelerates, the driver is pushed back into the seat due to the appearance of an inertial force. Even though an observer at rest outside the car does not feel this action directly, he becomes aware of its consequences by looking at the driver tossed toward the seat. In a sense, what we have found here is that the same happens for the radiation imbalance: it is indeed true that this imbalance can only be experienced by accelerated bodies through the emergence of a force opposite to the acceleration. Nevertheless, its existence can be inferred by any other inertial observer, the universal macroscopic manifestation being exactly what we call inertia.

Finally, we remark that a derivation of inertia involving Unruh effect has been proposed in Ref. [24] on the basis of Verlinde’s theory on the entropic origin of gravity and inertia [25]. However, this theory has been shown to possess some possible inconsistencies and its current version is not complete.

### Conclusions and outlook

Despite many attempts, the origin of inertia has not been adequately explained yet. In this paper, we have introduced a new model that traces the origin of this macroscopic property back to a more fundamental microscopic mechanism, *i.e.* the interaction among the resonant parts of Planck size of any body and the vacuum fluctuations around it. In this sense, our model is conceptually similar to Higg’s mechanism, where mass loses its status as a primary quality, becoming the result of elementary massless particles interacting with the Higgs field. Here, however, we stress that our considerations are applicable to elementary, as well as composite objects.

By employing the above prescription, we have presented two frameworks in which inertia can be easily modeled. The first one is based on the use of the Heisenberg Uncertainty Principle and the computation of the energy transferred by fluctuations to each resonant part of the body. The second approach is inspired by McCulloch’s theory of QI and ascribes inertia to the radiation imbalance due to the appearance of Rindler horizon (Rindler-scale Casimir effect). In spite of the underlying differences, the two frameworks are consistent as regards the resulting expressions for inertia. We emphasize that we have focused on the simplest analysis of a non-inertial motion with linear acceleration. Clearly, the case of a circular orbit is expected to be much more complicated, if only for the concerns on the existence of a rotational analogue of Rindler horizon (see Ref. [26] for a detailed discussion on this). It would also be interesting to address how our model of inertia is intertwined with the equivalence principle.

Some aspects remain to be addressed. For instance, in the context of a quantum description of gravity, several models predict a modification of the standard HUP to a Generalized Uncertainty Principle (GUP) which accounts for the emergence of the minimal length at Planck scale [27–35]. The question thus arises as to how this deformation is related to our result. In turn, a possible GUP-modified expression of inertia could allow us to put some bound on the GUP parameter

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have seen that each resonant part composing a given body contributes to its inertia through the interaction with fluctuations from any point of Planck volume of the quantized space. This entails that the ensuing effect is visible at macroscopic scale, being the sum of the interactions over the large number of resonant parts of the body.

through current limits on equivalence principle violations. On the other hand, in Refs. [36–39] possible non-thermal behaviors of the radiation perceived by accelerated observers have been pinpointed in various scenarios. In light of the established connection between such phenomenon and inertia, it would be worth studying whether the validity of our considerations is somehow affected by this deviation from thermality, and, if so, how it might be constrained by our analysis. More work is inevitably required along these and other directions.

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### References

- [1] Rovelli C. *J Am Philos Assoc* 2015;1:23.
- [2] Sciamia DW. *Mon Not R Astron Soc* 1953;113:34.
- [3] Dicke RH. *The theoretical significance of experimental relativity*. London: Blackie & Son; 1964.
- [4] Moon P, Spencer DE. *Philos Sci* 1959;26:125.
- [5] Brown GB. *Retarded action-at-a-distance*. Luton: Cortney Publications; 1982.
- [6] Assis AKT, Graneau P. *PRIRB-95*, 0000.
- [7] Haisch B, Rueda A, Puthoff HE. *Phys Rev A* 1994;49:678.
- [8] Unruh WG. *Phys Rev D* 1976;14:870.
- [9] Milgrom M. *Phys Lett A* 1999;253:273.
- [10] McCulloch ME. *Mon Not R Astron Soc* 2007;376:338.
- [11] Levin YS. *Phys Rev A* 2009;79:012114.
- [12] Renda M. *Mon Not R Astron Soc* 2019;489:881.
- [13] Benincasa DMT, Dowker F. *Phys Rev Lett* 2010;104:181301.
- [14] Dvali G, Gomez C. *Eur Phys J C* 2014;74:2752.
- [15] Casadio R, Giugno A, Orlandi A. *Phys Rev D* 2015;91:124609.
- [16] Chemisana D, Giné J, Madrid J. *Europhys Lett* 2020;130:60002.
- [17] Giné J. *Modern Phys Lett A* 2018;33:1850140.
- [18] Giné J. *Internat J Modern Phys A* 2017;32:1750043.
- [19] Modanese G. *Found Phys Lett* 2003;16:135.
- [20] McCulloch ME. *Europhys Lett* 2013;101:59001.
- [21] Giné J, McCulloch ME. *Modern Phys Lett A* 2016;31:1650107.
- [22] Smolin L. *Sci Am* 2004;290:66.
- [23] Giné J, Luciano GG. *Eur Phys J C* 2020;80:1039.
- [24] Lee JW. *Found Phys* 2012;42:1153.
- [25] Verlinde EP. *J High Energy Phys* 2011;04:029.
- [26] Crispino LCB, Higuchi A, Matsas GEA. *Rev Modern Phys* 2008;80:787.
- [27] Amati D, Ciafaloni M, Veneziano G. *Phys Lett B* 1987;197:81.
- [28] Maggiore M. *Phys Lett B* 1993;304:65.
- [29] Kempf A, Mangano G, Mann RB. *Phys Rev D* 1995;52:1108.
- [30] Amelino-Camelia G. *Internat J Modern Phys D* 2002;11:35.
- [31] Scardigli F. *Phys Lett B* 1999;452:39.
- [32] Scardigli F, Blasone M, Luciano G, Casadio R. *Eur Phys J C* 2018;78:728.
- [33] Buoninfante L, Luciano GG, Petruzzello L. *Eur Phys J C* 2019;79:663.
- [34] Luciano GG, Petruzzello L. *Eur Phys J C* 2019;79:283.
- [35] Luciano GG, Petruzzello L. *Eur Phys J Plus* 2021;136:179.
- [36] Marino J, Noto A, Passante R. *Phys Rev Lett* 2014;113:020403.
- [37] Blasone M, Lambiase G, Luciano GG. *Phys Rev D* 2017;96:025023; *J Phys Conf Ser* 2018;956:012021.
- [38] Carballo-Rubio R, Garay LJ, Martín-Martínez E, De Ramón J. *Phys Rev Lett* 2019;123:041601.
- [39] Blasone M, Lambiase G, Luciano GG, Petruzzello L. *Phys Rev D* 2018;97:105008; *Phys Lett B* 2020;800:135083; *Eur Phys J C* 2020;80:130.