preprint UdL

THE UNIQUE MIXED ALMOST MOORE GRAPH WITH PARAMETERS $k = 2$, $r = 2$ AND $z = 1$.

DOMINIQUE BUSET
Université Libre de Bruxelles,
Belgium.
dbuset@ulb.ac.be

NACHO LÓPEZ
Departament de Matemàtica, Universitat de Lleida
C/ Jaume II 69, 25001 Lleida, Spain.
nlopez@matematica.udl.es
http://www.matematica.udl.es/nacho

JOSEP M. MIRET
Departament de Matemàtica, Universitat de Lleida
C/ Jaume II 69, 25001 Lleida, Spain.
miret@matematica.udl.es

Received Day Month Year
Revised Day Month Year

A natural upper bound for the maximum number of vertices in a mixed graph with maximum undirected degree r, maximum directed out-degree z and diameter k is given by the mixed Moore bound. In this paper we prove that there is a unique mixed graph of diameter $k = 2$ and parameters $r = 2$ and $z = 1$ containing the largest possible number of vertices, which in this case is one less than the corresponding mixed Moore bound. Mixed graphs with prescribed parameters and order one less than the corresponding Moore bound are known as *mixed almost Moore graphs*.

Keywords: Moore graph; mixed graph; Diameter

1. Introduction

Network topologies based on mixed graphs arise in many practical situations, specially in those where the relationship between nodes (vertices) can be undirected or directed, depending on whether the communication is two-way or only one-way. Hence, a *mixed graph* G may contain (undirected) *edges* as well as directed edges (also known as *arcs*). From this point of view, a *graph* [resp. *directed graph* or *digraph*] has all its edges undirected [resp. directed]. The *undirected degree* of a vertex v, denoted by $d(v)$ is the number of edges incident to v. The *out-degree* [resp. *in-degree*] of v, denoted by $d^+(v)$ [resp. $d^-(v)$], is the number of arcs emanating from [resp. to] v. If $d^+(v) = d^-(v) = z$ and $d(v) = r$, for all $v \in V$, then G is said to be *totally regular* of degree d, where $d = r + z$. A *walk* of length $\ell \geq 0$ from u to v is a sequence of $\ell + 1$ vertices, $u_0 u_1 \ldots u_{\ell-1} u_\ell$, such that $u = u_0$, $v = u_\ell$ and each pair $u_{i-1}u_i$ for $i = 0, \ldots, \ell - 1$ is either an edge or an arc of G. A *directed walk* is a walk containing only arcs. An *undirected walk* is a walk containing only edges. A walk whose vertices are all different is called a *path*. The length of a shortest path from u to v is the *distance* from u to v, and it is denoted by $\mathrm{dist}(u, v)$. Note that $\mathrm{dist}(u, v)$ may be different from $\mathrm{dist}(v, u)$, when shortest paths between u and v involve arcs. The maximum distance between any pair of vertices is the *diameter* k of G. A *directed cycle* [resp. *undirected cycle*] of length ℓ is a walk of length ℓ involving only arcs [resp. edges] whose vertices are all different except $u = v$.

The *Degree/Diameter problem* for mixed graphs asks for the maximum number of vertices n in a mixed graph with maximum undirected degree r, maximum directed out-degree z and diameter k. This problem has been extensively studied both for the pure undirected case ($z = 0$) and for the pure directed case ($r = 0$), but there are fewer results for the general mixed case. The maximum number of vertices n for a mixed graph of diameter $k = 2$ with prescribed degree parameters r and z is upper bounded by

$$n \leq M(r, z, 2) = 1 + z + (r + z)^2$$

where $M(r, z, 2)$ is known as the *mixed Moore bound* of diameter 2 and it can be easily computed just by counting the maximum number of vertices at distance ≤ 2 from any given vertex in a mixed graph with maximum undirected degree r and maximum directed out-degree z. Mixed graphs of diameter k, maximum undirected degree $r \geq 1$, maximum out-degree $z \geq 1$ and order $M(r, z, k)$ are called *mixed Moore graphs*. The study of mixed Moore graphs was initiated by Bosak[1], but it has received much attention in the last decade (see Buset et al.[2], Nguyen et al.[8,9], Jørgensen[3], López et al.[4,5,6,7], etc.). It is known that mixed Moore graphs are totally regular of degree $d = r+z$ and they have the property that for any ordered pair (u, v) of vertices there is a unique trail of length at most k between them. The general problem of finding mixed graphs of maximum order given the parameters r, z, k remains unsettled, even for diameter $k = 2$. Bosák[1] gives the following necessary condition for the existence of a mixed Moore graph of diameter $k = 2$:

Theorem 1.1. Let G be a (proper) mixed Moore graph of diameter 2. Then, G is totally regular with directed degree $z \geq 1$ and undirected degree $r \geq 1$. Moreover, there must exist a positive odd integer c such that

$$r = \frac{1}{4}(c^2 + 3) \text{ and } c \mid (4z - 3)(4z + 5). \tag{1.2}$$

For those pairs (r, z) not satisfying Theorem 1, the maximum order of a hypothetical mixed graph in this context is $M(r, z, 2) - 1$, and these graphs are known as *mixed almost Moore graphs* of diameter 2. These extremal graphs were first studied recently by López and Miret[4].

Construction

The mixed Moore bound for diameter $k = 2$, undirected degree $r = 2$ and directed out-degree $z = 1$ states that the maximum order for a mixed graph with those parameters has at most $M(2, 1, 2) = 11$ vertices. Such a graph does not exist because the pair $(r, z) = (2, 1)$ does not satisfy the conditions given in Theorem 1. Hence, such an extremal graph in the context of the degree/diameter problem, should have at most 10 vertices. A mixed graph G of order 10, diameter 2, maximum undirected degree $r = 2$ and maximum directed out-degree $z = 1$ can be constructed as follows:

The vertex set of G is the union of the two sets U and U', where $U = \{u_i \mid i \in \mathbb{Z}_5\}$ and $U' = \{u'_i \mid i \in \mathbb{Z}_5\}$. Then, the edges are defined by the symmetric relationships $u_i \sim u_{i\pm1}$ and $u'_i \sim u'_{i\pm2}$, where the subscripts operations are taken over $\mathbb{Z}_5$. Besides, the arcs are defined by $u_i \to u'_{i+1}$ and $u'_i \to u_{i+1}$ (see Fig. 1).

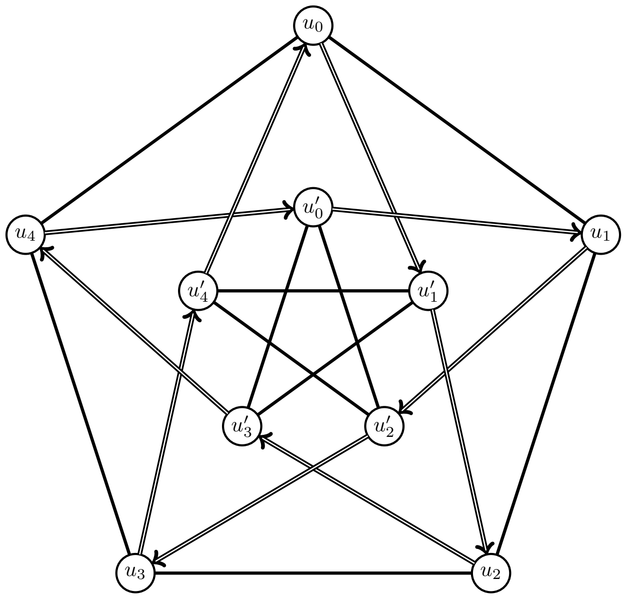


Fig. 1. The unique mixed almost Moore graph of diameter $k = 2$ and parameters $r = 2$ and $z = 1$.

This mixed graph G is totally regular, but it is not vertex-transitive. The automorphism group of G is the permutation group generated by $(u_0\ u_1\ u_2\ u_3\ u_4)(u'_0\ u'_1\ u'_2\ u'_3\ u'_4)$, that is, every automorphism of G is one of the five rotations of the outter pentagon and inner star that keep the symmetry of the Fig. 1. Another way to construct G is by voltage assignment, using a dipole as basis graph and $\mathbb{Z}_5$ as basis group. G was first introduced by López and Miret[4] where the characteristic polynomial of any totally regular mixed almost Moore graph of diameter 2 is computed. In particular, for $r = 2$ and $z = 1$, it is shown that such characteristic polynomial must be

$$\phi(x) = (x-3)(x+2)^{a_1}(x-1)^{b_1}x^{a_2}(x+1)^{b_2}\prod_{i=3}^{10}\Phi_i(x^2+x-1)^{\frac{m(i)}{2}},$$

where a_i and b_i are positive integers and $m(i) = \sum_{i|l} m_l$, where $(m_1, \ldots, m_{10})$ is the permutation cycle structure of the graph. The characteristic polynomial depends on some cyclotomic polynomials denoted by $\Phi_i(x)$. It can be seen that those parameters must satisfy some hard restrictions in this case, like $(a_2 - b_2) - 3(a_1 - b_1) = 3$, $m_i = 0, \forall i > 5$, etc. from where one can restrict the existence of $\phi(x)$ for just six cases. One of them ($a_1 = a_2 = b_2 = 0, b_1 = 1$ and $m_i = 0\ \forall i \neq 5$, $m_5 = 2$) gives the characteristic polynomial of G, which is

$$\phi_G(x) = (x-3)(x-1)\Phi_5(x^2+x-1).$$

Although this spectral approach would give us just one possible characteristic polynomial for a mixed almost mixed Moore graph, this result does not provide a proof of the uniqueness of G, since other cospectral graphs could be non isomorphic to G. Next section will show that G is in fact unique, that is, there is only one mixed almost Moore graph of diameter 2 and parameters $r = 2$ and $z = 1$.

Uniqueness

Let G be a mixed graph of 10 vertices, diameter 2, maximum undirected degree $r = 2$ and maximum directed out-degree $z = 1$. In this section we will show that G must be unique. Our proof proceeds by building up the arcs and edges of the graph in a systematic fashion. At any point in this construction, we say that a vertex v of G is *full-edges* [resp. *full-arcs*] if all the extremities of the edges [resp. arcs] emanating from v are known. A vertex v of G is called *complete* if it is full-edges and full-arcs.

Let us start with a useful lemma that will provide us some restrictions on the subgraphs that may exist in G. We notice that for such mixed graph G, every vertex must be incident to exactly two edges and one outcoming arc, since otherwise it is not possible to reach all the remaining vertices at distance ≤ 2 from a given vertex. Nevertheless, it is not clear that this must be true for the incoming arcs, that is, such graph might not be totally regular a priori. Hence, we do not assume such regularity for the in-degree in the following results.

Lemma 1.1. *Let G be a mixed almost Moore graph of diameter 2, maximum undirected degree $r = 2$ and maximum directed out-degree $z = 1$. Then,*

(a) There is no edge at distance 1 from any vertex of G.
(b) Every arc of G is included either in $\overrightarrow{C}_3$ (directed cycle of length 3) or in a $\overrightarrow{C}_3$ with an arc replaced by an edge.
(c) G has no subgraph isomorphic to T, where T is a triangle having a vertex u with two incoming arcs, and the remaining two vertices sharing an edge.

(d) G has no subgraph isomorphic to C_4, that is, the (undirected) cycle graph of length four.

Proof.

(a) If there exists an edge at distance 1 from a vertex v of G, then, due to the restrictions on the maximum undirected degree, the maximum directed degree and the diameter, G has at most 9 vertices (see Fig. 2), which is impossible.

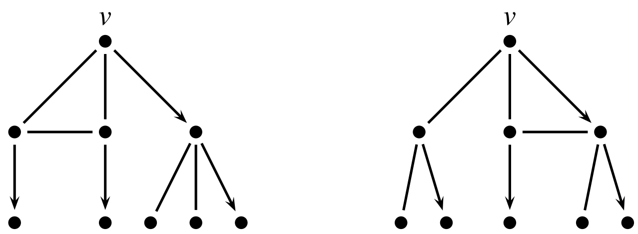


Fig. 2. The two possible situations when G has an edge at distance 1 from any given vertex v. In both cases the number of vertices of G is at most 9.

(b) For any arc (u, v) of G, there must exist a trail of length 2 from v to u (since the diameter of G is 2). Let v, w, u be such a trail, where the relationship $v \sim w$ and $w \sim u$ could be, a priori, either an arc or an edge. By (a), $v \sim w$ and $w \sim u$ can not be both edges at the same time, so (u, v) is included either in $\overrightarrow{C}_3$ or in directed cycle of length 3 with an arc replaced by an edge.

(c) Suppose T is a subgraph of G. Since G has 10 vertices, G must contain the subgraph depicted in Fig. 3, where T is the subgraph of G induced by the vertices $\{v_0, v_2, v_3\}$. From here on, we label the remaining vertices of G as in the figure. Since the diameter of G is 2, there must exist a trail of length 2 from v_3 to the vertices v_0, v_1 and v_2, which are all full-edges. So, there must be an arc from one of the vertices into the set $\{v_7, v_8, v_9\}$ to v_0 [resp. v_1 and v_2]. On the other hand, by (a), there is no edge between two of the vertices into $\{v_7, v_8, v_9\}$. Therefore, the 4 remaining edges incident to one of the vertices of $\{v_7, v_8, v_9\}$ must have their extremities in $\{v_4, v_5, v_6\}$. In particular, there is an edge joining v_6 and one of the vertices of $\{v_7, v_8, v_9\}$, and then v_6 is full-edges. However, there must be a trail of length 2 from v_2 to v_4 (resp. v_5). These two paths must contain v_6 and then there must be an arc (v_6, v_4) and an arc (v_6, v_5), which is impossible.

(d) Suppose such a subgraph exists in G. Since G has 10 vertices, G must contain the induced subgraph depicted in Fig. 3, where C_4 is the subgraph induced by $\{v_0, v_1, v_2, v_5\}$. Moreover, since the diameter of G is 2, there is a trail of length 2 from v_3 to every vertex in $\{v_0, v_1, v_2\}$. So, there must be exactly one arc from all of the vertices in $\{v_7, v_8, v_9\}$ to one vertex in $\{v_0, v_1, v_2\}$.

However, since there must exist a trail of length 2 from v_3 to v_5, one vertex belonging to $\{v_7, v_8, v_9\}$ must be adjacent to v_5 by an edge (because vertices v_7, v_8 and v_9 are full-arc) which is a contradiction because v_5 is full-edges. □

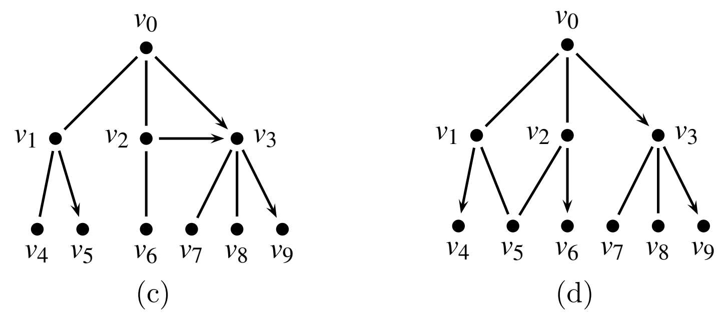


Fig. 3.

Now, we are ready to prove the main result of the paper.

Theorem 1.2. There exists a unique mixed almost Moore graph of diameter 2, maximum undirected degree $r = 2$ and maximum directed out-degree $z = 1$.

Proof. Let $v_0 \in V$ be any vertex of a mixed almost Moore graph G of diameter 2 and parameters $r = 2$ and $z = 1$. Lemma 1.1 gives us two cases, corresonding to whether there exists an arc at distance 1 from v_0 or not. Let us denote by v_1 and v_2 those vertices joined by an edge to v_0 and v_3 the unique vertex such that (v_0, v_3) is an arc.

Case I: Suppose there is an arc at distance 1 from v_0. By symmetry and Lemma 1.1 (a) and (c), it is not restrictive to suppose that (v_3, v_2) is such arc. Since G has 10 vertices, and due to the restrictions on the degree and the diameter, the graph depicted in Fig. 4 must be a subgraph of G. From here on, we follow the labeling of the vertices depicted in the figures.

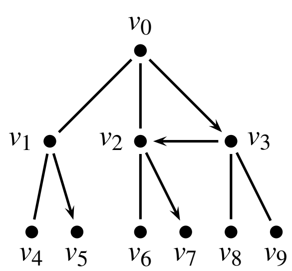


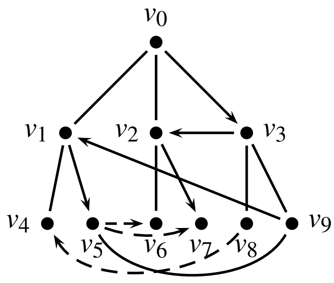
Fig. 4.

Since v_3 must reach v_1 in two steps, there must be an arc from v_8 or v_9 to v_1 (note that v_3 is already complete and v_1 is full-edges). By symmetry, we can assume that (v_9, v_1) is such arc. By Lemma 1.1 (a) and (c), vertex v_9 cannot be adjacent to v_4, and then v_8 must be adjacent to v_4 by an arc or an edge (v_3 must reach v_4 in two steps). By the various parts of Lemma 1.1, the remaining edge from v_9 cannot be to any of $\{v_0, v_2, v_4, v_8\}$, so, there is an edge $v_9 \sim x$ where $x \in \{v_5, v_6, v_7\}$. If $x = v_6$, then v_6 is full-edges and then, to reach v_7 from v_9, the only possibility would give the arc (v_6, v_7) which is impossible by Lemma 1.1 (c). Therefore, $x \in \{v_5, v_7\}$ which gives us two possibilities for the last edge incident to v_9:

(Ia) Assume $v_9 \sim v_5$ is an edge (see Fig. 5). The existence of this edge implies that v_5 must be adjacent to v_6 and v_7. Then, the only possibility to reach v_8 from v_1 by a trail of length 2 is through v_4. So, there is an edge between v_4 and v_8 (we already know that v_8 is adjacent to v_4). Then, to reach v_0 from v_5 by a 2-trail, (v_7, v_0) must be an arc of G (lemma 1.1 (a), and vertex v_1 is already complete). Now, vertices v_0 and v_2 are complete, v_8 is full-edges and v_7 is full-arc, so a 2-trail from v_2 to v_8 must contain the arc (v_6, v_8). Every vertex of G is full-edges with the exception of v_6 and v_7. Moreover, v_7 is full-arc, and its two incident edges are still unknown. It follows that the only possibilities for these two edges are $v_7 \sim v_5$ or $v_7 \sim v_6$, which implies that (v_5, v_6) must be an arc, but this is a contradiction with Lemma 1.1 (a) (see the triangle with vertices v_5, v_6, v_7).

(Ib) Suppose $v_9 \sim v_7$ is an edge (see Fig. 5): Then the vertex v_9 is complete. To join v_3 to v_5 by a 2-trail, then v_8 must be adjacent to v_5 (v_4 is not adjacent to v_5 and v_2 is complete). The same argument applies to v_4. Therefore, v_8 is complete, and either (v_8, v_5) is an arc (and then $v_8 \sim v_4$ is an edge) or (v_8, v_4) is an arc (and $v_8 \sim v_5$ is an edge). So, at this point we have that v_8 is connected to both v_4 and v_5. The 2-trail from v_8 to v_0 gives us a new arc which must be (v_5, v_0). Again, the 2-trail from v_7 to v_0 gives us a new edge which must be $v_7 \sim v_5$. We also must have a 2-trail from v_8 to v_6 which must pass through v_4. But since v_4 must be at distance ≤ 2 from v_2, then the adjacency between v_4 and v_6 must be realized by the edge $v_4 \sim v_6$ (v_0, v_2 and v_7 are complete). Moreover, since v_6 is full-edges, the 2-trail from v_2 to v_8 must contain the arc (v_6, v_8). Therefore, v_4 is full-edges and the connection between v_8 and v_4 cannot be an edge and thus is an arc (v_8, v_4) which implies that $v_8 \sim v_5$ is an edge. To complete the graph, it suffices to note that to reach v_4 to v_3 by a 2-trail, the only possibility is to add the arc (v_4, v_9). Then, the mapping $v_0 \mapsto u'_0$, $v_1 \mapsto u'_3$, $v_2 \mapsto u'_2$, $v_3 \mapsto u_1$, $v_4 \mapsto u'_1$, $v_5 \mapsto u_4$, $v_6 \mapsto u'_4$, $v_7 \mapsto u_3$, $v_8 \mapsto u_0$ and $v_9 \mapsto u_2$ between the set V of vertices of G and the set $U \cup U'$ described in Section 1 gives an isomorphism between the graph obtained here and the graph depicted in Fig. 1.

So, in case I, G must be isomorphic to the already known graph. It follows that



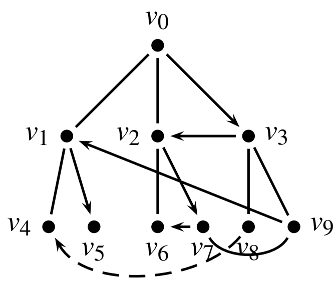


Fig. 5. The two cases derived from the first case of the proof. Dashed lines mean that such connection could be either an arc or an edge.

in Case II, we may assume that there is no oriented triangle with at least one edge. We will see that in this case G cannot exist.

Case II: Assume there is no arc at distance 1 from v_0, then, to reach v_0 from v_3 by a 2-trail, there must exist a directed cycle v_3, v_7, v_0. Since v_3 must reach v_1 [resp. v_2], then one vertex of $\{v_7, v_8, v_9\}$ must be adjacent to v_1 [resp. v_2]. These two adjacencies cannot be done simultanously by edges, since otherwise $n \leq 9$, so at least one of them is an arc, and by symmetry we can assume that this arc is (v_8, v_2) (v_7 is already full-arc). If the 2-trail from v_3 to v_1 passes through a vertex x, then $x \sim v_1$ is either an arc and then $x = v_9$, or an edge and then $x \in \{v_8, v_9\}$ (note that Lemma 1.1 implies $x \neq v_7$). Let us examine those two cases:

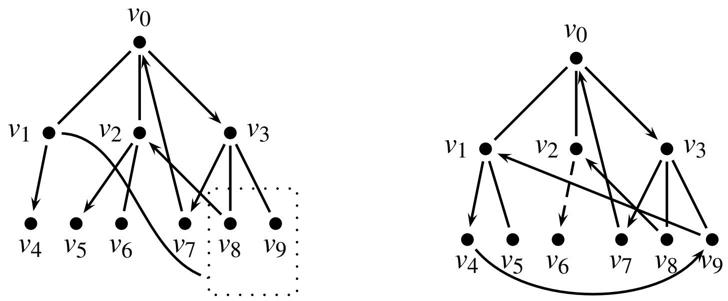
(IIa) Let $x \sim v_1$ be an edge. Since $n = 10$, it follows that the remaining arcs and edge starting at v_1 and v_2 cannot be incident to $\{v_7, v_8, v_9\}$, and it is not restrictive to assume that (v_1, v_4), (v_2, v_5) are the arcs and $v_2 \sim v_6$ is the edge (see Fig. 6). However, there must exist a 2-trail from v_7 to v_8 [resp. v_9], and by Lemma 1.1 (a), v_7 (which is full-arc) is not adjacent to either v_8 or v_9. Therefore, there is one vertex y such that $v_7 \sim y$ and y connects to v_8. The vertex y cannot be v_6, because it would imply the existence of the arc (v_6, v_8) which lead us to case I, already treated. So $y \in \{v_4, v_5\}$. If $x = v_8$, that is, $v_1 \sim v_8$ is an edge, then (y, v_8) is an arc, which implies that $y = v_5$ (to avoid the already treated case I with arcs (v_1, v_4), (v_4, v_8) and edge $v_8 \sim v_1$) and $v_7 \sim v_5$ must be an edge of G. But then, we must connect v_1 to every vertex into the set $\{v_5, v_6, v_7, v_9\}$ by 2-trail which must pass through v_4 (v_1, v_0, v_8 are complete). However, we can connect v_4 to at most 3 vertices and not 4 which gives us a contradiction. Hence, $x = v_9$, that is, $v_1 \sim v_9$ is an edge of G. Therefore, there is a vertex $y' \in \{v_5, v_6\}$ such that $v_7 \sim y'$ and y' connects to v_9 (note that $y' = v_4$ falls into case I with the arcs (v_1, v_4), (v_4, v_9) and the edge (v_1, v_9)). So v_9 is full-edges and we already knew that $y \in \{v_4, v_5\}$. Therefore, to go from v_9 to v_2 by a 2-trail, the unique out-arc from v_9 must have its extremity in $\{v_2, v_6\} - \{y'\}$ (v_0, v_4 and v_8 are eliminated because of Lemma 1.1 (a) and (c) and Case I). But if (v_9, v_6) is an arc, then $y' = v_5$ which shall be full-arc and so in order to reach v_9 to v_5 by a 2-trail, we must have an edge between v_5 and one of the vertices in $\{v_1, v_3, v_6\}$ which is a contradiction. It follows that (v_9, v_2) must be an arc of G. In order to avoid the case already treated, we get $y' \neq v_6$ which implies $y' = v_5$, that is, we have the edge $v_7 \sim v_5$. But then, we must connect v_1 to vertices $\{v_5, v_6, v_7, v_8\}$ by 2-trails which must pass through vertex v_4, and there are only 3 connections, which gives us a contradiction for this case.

(IIb) Let (x, v_1) be an arc. This arc must be (v_9, v_1). Vertices v_4, v_5, v_6 must have a connection coming from v_1 and v_2. We can assume that the arc and an edge from v_1 are (v_1, v_4) and $v_1 \sim v_5$, respectively, and $v_2 \sim v_6$ with an arc or an edge. Moreover, since v_9 must be at distance 2 from v_1, there must exist an arc (v_4, v_9). Also, there must exist a vertex $x \in \{v_4, v_5, v_9\}$ joined with an edge or an arc with origin v_2 (the other vertices are impossible because of Lemma 1.1). Again we have two cases to take into account:

If $v_2 \sim x$ is an edge, then $x \in \{v_4, v_9\}$ and (v_2, v_6) is an arc. However, if $v_2 \sim v_4$ is an edge, then v_1 must reach respectively v_6, v_7, v_8 by 2-trails which must contain the 3 remaining unknown arcs and edges from v_4 and v_5. By Lemma 1.1 the unique edge with v_4 as origin, must be $v_4 \sim v_7$, and then the arc/edge with origin v_5 must have for extremities v_6 and v_8. Since v_8 must be related to v_1 by a 2-trail, the only edge from v_8 to construct this 2-trail must be $v_8 \sim v_5$ and then (v_5, v_6) must be an arc. It follows that the two last edges issue from v_6 must have for extremities v_7 and v_9, respectively (there is no edge between v_7 and v_9, and all the other vertices are full-edges). Therefore, there is no possibility to reach v_5 from v_2 by a 2-trail, which is a contradiction. Since $v_2 \sim v_4$ cannot be an edge of G, the vertex $x = v_9$ is now complete. However, v_2 must be connected to v_4, v_5, v_7, v_8 by 2- trails which must all pass through v_6, and this is again impossible.

If $v_2 \sim x$ is an arc, then $v_2 \sim v_6$ is an edge. In order to connect v_2 to v_8 by a 2-trail, $x \sim v_8$ must be an arc, since otherwise the arcs (v_2, x), (v_8, v_2) and the edge (x, v_8) would take us to case I, already treated). v_4 and v_9 are already full-arc, so the only possibility is $x = v_5$. There must exist a 2-trail from v_1 to reach v_6, and then we must add the edge $v_4 \sim v_6$ (v_5 cannot be used because we would be in case I already treated). Then, there is no possibility for the last edge incident to v_9, which leads us to a final contradiction.

□



Case (IIa) Case (IIb)

Fig. 6. The two cases derived from the second case of the proof. Dashed lines means that such connection could be either an arc or an edge.

Acknowledgments

To our beloved friend Mirka Miller. She would love to see this result concerning mixed Moore graphs. Research of Nacho López and Josep M. Miret was supported in part by grants MTM2013-46949-P (Spanish Ministerio de Economa y Competitividad) and 2014SGR-1666 (Generalitat de Catalunya).

References

1. J. Bosák. Partially directed Moore graphs. *Math. Slovaca*, (29):181–196, 1979.
2. D. Buset, M. El Amiri, G. Erskine, M. Miller, and H. Pérez-Rosés. A revised Moore bound for mixed graphs. *Discrete Mathematics*, 339:2066 – 2069, 2016.
3. L. K. Jørgensen. New mixed Moore graphs and directed strongly regular graphs. *Discrete Mathematics*, 338(6):1011 – 1016, 2015.
4. N. López and J.M. Miret. On mixed almost moore graphs of diameter two. *The Electronic Journal of Combinatorics*, 2(23):1 – 14, 2016.
5. N. López, J.M. Miret, and C. Fernández. Non existence of some mixed Moore graphs of diameter 2 using SAT. *Discrete Mathematics*, 339(2):589 – 596, 2016.
6. N. López, H. Pérez-Rosés, and J. Pujolàs. Mixed Moore Cayley graphs. *Electronic Notes in Discrete Mathematics*, 46:193 – 200, 2014.
7. N. López and J. Pujolàs. Properties of mixed Moore graphs of directed degree one. *Discrete Mathematics*, 338(4):522 – 526, 2015.
8. M. H. Nguyen and M. Miller. Moore bound for mixed networks. *Discrete Mathematics*, 308(23):5499 – 5503, 2008.
9. M. H. Nguyen, M. Miller, and J. Gimbert. On mixed Moore graphs. *Discrete Mathematics*, 307(78):964 – 970, 2007.