Modelling tactical planning decisions through a linear optimization model in sow farms

Sara V. Rodríguez-Sánchez¹², Lluís M. Plà-Aragonés¹* and Victor M. Albornoz³

¹ Departmento de Matemàtica. Universidad de Lleida, Campus Cappont, c./Jaume II, 73, 25001 Lleida, España.
⁻³ Departamento de Industrias, Universidad Técnica Federico Santa María, Campus Santiago, Av. Santa María, 6400 Santiago, Chile

* Corresponding author
Abstract

This paper deals with tactical planning decisions for breeding farms producing piglets through a linear optimization model. A medium-term planning horizon based on weekly periods is considered. The proposed model maximizes the profit of the farm and takes into account sow herd dynamics, housing facilities, reproduction management, available stocks and a target quota of a weekly number of farrowing to integrate piglet production into the pig supply chain management. As result, an optimal replacement policy of sows and a scheduling of purchases of gilts during the whole planning horizon are provided. The model is solved using the algebraic modelling language ILOG OPL 6.1 with CPLEX 11.2 as the linear optimization solver. The article also discusses results obtained from a sensitivity analysis performed to assess the suitability of the model approach and the benefits of representing real variability over time against time homogeneity. In addition, the analysis and results presented lead to better understand sow herd dynamics over a finite time-horizon and corresponding performance.

Keywords: Tactical decisions; Planning; Sow herd management; Replacement problem; linear programming;

1. Introduction

During the last decades, a noticeable change in the structure of the European pork sector has been observed (Trienekens et al., 2009). In Spain for instance, pig production has evolved from small scale-family operated farms to an industrial structure which is characterized by a production concentrated in larger and more specialized pig production units. In most cases usual units are for instance breeding farms producing piglets, rearing farms producing young pigs and fattening farms producing pigs to be slaughtered. Actual pig production is the result of the integration of several of these specialized production units under a pork supply chain (PSC) framework (Rodríguez et
This requires the coordination and cooperation between production phases usually done by companies or cooperatives having the control over production in a set of farms. In this context, most of the pigs are produced in farms owned by a big company or by independent farmers who sell all pig production by contract to a company or cooperative usually called integrator (see Ouden et al., 1996). Thus, the success of the pig production relies on a good pork supply chain management supported by an accurate individual farm management (Vorst et al., 2007).

Breeding farms devoted to piglet production are more complex to manage than rearing or fattening farms. Piglet production is intimately related to the reproduction process of sows and many different factors other than feeding may affect the final results. Furthermore recent EU regulations concerning pig welfare (affecting for instance housing facilities for sows or fixing a minimum lactation period) reduced or bound the margin of benefit that individual farms could have attained years ago. Hence, the increasing number of new variables and constraints affecting piglet production make difficult to explore all possible management alternatives to find the best one. Therefore, optimization models have to play an even more important role in modern farm management, improving the quality of farm management and enhancing the competitive position of farms. For such a purpose the development of good models is important, but also the mathematical knowledge required exploiting and interpreting model outcomes in a practical context.

Decisions on farm are taken at operational, tactical and strategic levels as discussed by Jalvingh et al. (1992). Strategic decisions on sow breeding farms have been well covered by research studies and several models have been developed to support this
type of decisions as was pointed out by Plà (2007). They are suitable tools to support sow herd productivity assessment, evaluating sow herd performance or to analyse alternative herd management strategies (Upton, 1989). Nevertheless, some important details with practical relevance for weekly operations on farm had been left aside. For instance, seasonal variations of fertility, limited supply of gilts or a target for the number of farrowings in connection with the capacity of lactation facilities. A first attempt to fill this gap at operational level has been made by Martel et al. (2008). They proposed a simulation model paying attention to the distribution of periodic task events and derived performances in sow farms operating under different batch farrowing systems (BFS). Finite time horizon models seem to be a better option to support decision making tasks in weekly operations, as it is going to be confirmed in this paper.

Reasons for not including these aspects in past models were related to the lower computational power of computers, the complexity of the system and some mathematical shortcomings of optimisation approaches.

Different operations are performed on a breeding farm, most of them after having grouped animals in batches and affected by the PSC context (i.e. integrator commitments). In general, a weekly basis period is adopted to rationalise the daily work on sow farms when BFS is adopted (Martel et al., 2008). Also, purchases of gilts and culling of animals are not effective daily. Culled animals are also grouped one day a week to be transported to the slaughterhouse and replaced by young gilts. These replacement gilts are generally purchased a few weeks before they are inseminated. In this study it is assumed the existence within the PSC of an external quarantine unit, acting like a gilt supplier, capable to serve all ordered gilts. This fact is quite often in big companies or cooperatives in Spain and because of that quarantine is not considered
part of this model. The sow replacement decision is especially important in piglet production activity because it determines future productivity of the herd. This is so because along the age-structure of the herd, gilts and old sows are less productive than young or medium age sows. In addition, the age-structure of the herd may affect the sensibility to disease outbreaks or passive immunization by contact between young and mature sows. On the other hand, a fixed scheduling of replacement and purchases have no sense in the long term because the dynamics in a commercial sow herd is variable (regular annual replacement rates are around 50%, but on average). Also, seasonal variations in reproductive performance and pig meat demand are frequently observed and may strongly affect herd dynamics and net revenues impacting the replacement policy. Then, the scheduling should be adapted depending on actual state of the herd and expected market conditions. When scheduling replacements, farmers try to take into account future possible variations, and hence the scheduling is for the medium term, i.e. periods under a year. Also it is very important to point out that globalization makes piglet production connected with the production of other agents of the PSC, in particular when they are part of a vertical integration scheme. In view of that, it is common to have a target quota of weaning or farrowing established by the integrator to attain a steady flow of pigs through the PSC.

The objective of this paper is to formulate a Linear Programming model for scheduling replacements and purchases week by week in sow farms producing piglets over a finite time horizon. Besides that, understanding the impact of replacements and purchases on sow herd dynamics and variables related. Replacement and purchase decisions are two of the most important tactical decisions on sow farms (Dijkstra et al., 1986; Huirne et al., 1993). These decisions are sensitive to changes from time to time due to variations
in prices or in reproductive performances of sows. In this sense, sensitivity analyses are performed to assess the importance of some productive and economic parameters as well as derived impact on herd dynamics. Changes in parameters (or decisions) do not only affect actual results but also determine herd structure over time and consequently future production. The model is tailored to specific individual farm conditions in the context of a PSC (i.e. an unrestricted supply of gilts, medicines, insemination doses and feedstuffs on demand, by fixing a target quota of farrowings and the selling of all piglets produced). It includes the productive and reproductive behaviour of a group of breeding sows over time where piglets are the commercial product. Hence, the herd model is mainly focused on reproduction and replacement management of sows. The objective function maximizes profits constrained in different aspects like the purchase of gilts, the replacement of sow or the efficient occupancy of facilities according to reproduction performances and farrowing goals (given that farrowing facilities are the most expensive in sow farms).

2. The linear optimization model

Several optimisation models have been published up to now. Dynamic programming or Markov Decision Processes (MDP) have been the methods preferred (Plà, 2007). It is well known that several algorithms are available to solve a MDP, one of them is based on LP (Puterman, 1996). However, linear programming (LP) had not been used as much as in other disciplines, in part due to the complexity of the system which led to unsolvable problems given the computational limitations at the time. Although LP is not the most efficient algorithm to solve a MDP, it presents nowadays some advantages that make it interesting for practical use, such as the existence of powerful solvers, the
flexibility to add constraints and the capability of extending the model to more complex
stochastic models (Rodriguez et al., 2009).

The proposed linear programming model will mainly try to represent the sow herd
dynamic behaviour (see Figure 1) to determine the optimal purchase and replacement
policy for a given planning horizon. It considers a medium term planning horizon,
divided into a set $T$ of weekly periods and maximizes the total profit of the production
plan, $\Phi$, while satisfying a set of constraints that mainly concern the sow herd dynamics
behavior and the farm capacity (see the Appendix). Profit is defined as the difference
between income obtained by sales and the different costs of production incurred.

Income is generated from two sources. The first one corresponds to sales of piglets
weaned per sow depending on period and reproductive cycle. The second source of
income comes from sales to the slaughterhouse of culled sows. Each culled sow has a
different selling value regarding individual live weight which is affected by cycle and
reproductive state. Culled sows include also sows with abortion. Production costs
considered are feeding cost of sows and piglets, labour, insemination, veterinary
expenses and replacement gilts. These costs are calculated per period and summarised
per sow (€/head/week) being in gestation, lactation, waiting for pregnancy control or
waiting for insemination. The occupation of housing facilities is also taken into account.

Normally, Spanish sow farms have three different types of facilities: breeding-control,
pregnancy and farrowing facility (Plà et al., 2009). Breeding facility is where sows are
inseminated and controlled in order to confirm the pregnancy (around three weeks after
the insemination). Once the pregnancy is positively confirmed sows are moved to the
pregnancy facility. Otherwise, it is considered that conception has failed and they
remain in the same facility for a subsequent re-insemination, according to a maximum
number of attempts that is part of the management policy. Farrowing facility is where farrowing and weaning operations are done. So, before farrowing (normally one week), pregnant sows are moved to the farrowing facility and sows remain there until weaning (normally 3 weeks after farrowing). This representation considers different reproductive cycles (weaning to weaning interval or purchase to weaning interval only for the first one) or parities in the sow lifespan, assuming that at the end of it a sow is sold to the slaughterhouse and replaced by a purchased gilt (see Figure 1). A quarantine unit is also rather common to introduce progressively new gilts on farm. However, big companies operating as PSC tend to concentrate these units as a separate one acting as a gilt supplier and getting a better control on the health status over the whole PSC. Therefore, quarantine is not considered in our sow herd model.

The previous objective function allows choosing the better feasible herd management strategy among all the feasible solutions satisfying a set of constraints. These constraints enumerated from (2)-(30) are formulated mathematically in Appendix A. Constraints can be grouped as follows:

- **Initial conditions**, they account for the initial herd distribution of sows over different states at \( t=1 \) (2)-(5). They can be adapted to any particular situation either a starting farm or an existing one. The values are computed apart and tend to the herd structure at equilibrium or long term herd structure, according to some complementary study and methodology used by the authors (Plà et al., 2009).

- **Herd dynamics** over time describes all possible biological transitions of sows evolving from one state to another. They are represented in constraints (6)-(13), (27), (28). They represent the inter-temporal behaviour of sows under different replacement policies. Constraint (27) refers to gestating sows suffering an
abortion according to an abortion rate, $\alpha_{rt,c,g}$, (for modelling purpose we assume these are concentrated in week 13 of gestation). They are culled thereafter of the herd given that abortion is a common culling reason often adopted by pig specialists in Spain. Constraint (28) represents mortality and it is defined as a rate of sows that not longer survive where $\alpha_{r't,c,g}$ represents the mortality rate per cycle, in the $g^{th}$ week of gestation at period $t$ (notice that $\alpha_{r,c,g} + \alpha_{r',c,g}$ must be equal to $1-\alpha_{r,c,g}$).

- **Capacity**, the model presents a set of constraints related to the maximum capacity (in number of sows) of each of the considered three facilities: service (14), pregnancy (15) and lactation facilities (16).

- **Smooth variations** in purchases from week to week and fixing bounds for the minimum and maximum number of gilts allowed to be purchased per period (17) - (18).

- **Final conditions**, they represent the ending stock of animals, representing the continuity of the farm beyond the end of the considered finite time horizon (19)-(22). They are stated based on the initial stock just to represent the stability of the herd.

- **Voluntary Culling conditions**, they represent situations at specific states where sows will be voluntary removed from the herd as constraints (23)-(26) represent. For instance, when reaching the maximum number of inseminations (23) or number of reproductive cycles (24), sows are culled. It is assumed only animals in the breading and lactation facilities can be culled voluntary and done before being transferred to the next facility. And constraints (25) and (26) were set to bound the total number of culled sows at the end of the planning horizon $T$. 
- **Age structure Stability**, in order to keep the age structure of the population stable by usual replacement rates, a minimum number of culled sows (voluntary and involuntary) \( \varphi \) was set and represented by constraint (29).

- **Target of farrowings** (30). Usually this target is specified indirectly by an agreement between the farmer and the integrator through a contract. It is related to the reproductive performance, batch management and the committed number of piglets weaned per week. Piglets weaned per week can also be seen as a demand constraint in the model related to the capacity of farrowing facilities. Furthermore these constraints are only considered beyond an initial subset of periods \( T_1 \), assuring the fulfilment of the target quota.

The piglet production model further contributes to decide in which cycle a sow should be culled, as well as in which state inside the cycle. It indicates how many gilts must be purchased to meet targets of farrowing or weaning quota. A target allows a better coordination and management of piglet production into large PSC. Herd size need not be constant as most of the models published until now require (Plà, 2007). It was expected the model leads to a maximum use of farrowing facilities since the economic rewards are mostly concentrated in farrowing stage.

### 3. Parameters and Scenarios

In this section data sources and values for model parameters are presented. Furthermore, a set of scenarios are defined in order to illustrate the suitability and advantages of the proposed optimization model. Basic parameters such as conception rate, litter size, mortality rate and abortion rate were estimated using the maximum likelihood method. The statistical data were taken from standard values under Spanish conditions and
recorded in the BD-Porc databank (national record keeping system hosted at http://www.irta.es/bdporc/, accessed 16 February 2010), and do not correspond to a specific farm. While some others parameters were taken from the literature. In order to generate a more realistic economic environment in contrast of using average economic parameters, different historical Spanish series of prices were considered. The algebraic modelling language ILOG OPL 6.1 was used with CPLEX 11.2 as the linear optimization solver for implementing and solving the different instances developed.

3.1. Sources of data and parameters

In this study an initial herd size of 2,170 sows was considered a regular size to represent a big commercial Spanish sow herd and it is considered to represent a farm under an industrial structure (i.e. PSC). As stated, the model requires setting the initial and final distribution of sows over the different reproductive states. Hence, to represent the stability of the herd, initial and final herd distribution was assumed to be the same and selected from the steady-state distribution. That was derived from the basic case running under an infinite time horizon related with (19)-(22) constraints, using the model of Plà et al. (2009). A maximum number of 12 parities were allowed as sow lifespan. Parity was considered to finish either with a weaning or with an abortion. The actual capacity of the lactation facility was of 500 crates. Hence, a rate of 100 farrowing/week represented a measure of the weekly work load expected by the farmer and a goal to be met by the farm according to vertical integration requirements. The maximum number of allowed inseminations was three, beyond that; infertility was considered a reason of voluntary culling, just like an abortion. Although the model was solved assuming a planning time horizon of $T=156$ weeks (approximately 3 years) only results concerning the first 52
weeks (a year) were considered. This way, occasional perturbations caused by the lack
of information beyond the end of the time horizon (i.e. final inventory) are avoided.

Parameters regarding fertility are described in Table 1. Nevertheless the conception rate
decreases on warm periods. It is assumed based on expert advice a decrease of 3% in the
months of June and September and 5% in the hottest months of July and August.
Mortality rate during gestation period was around 2% (all casualties modelled together
at the 16th week), while abortion rate was around 1% (modelled all abortions at the 10th
week). Mortality was also concentrated in summer time, distributing the 70% of
casualties in summer (June to September) and the 30% the rest of the year. The
minimum number of culled sows (voluntary and involuntary) was set to 20 per week in
order to keep population stable by usual replacement rates around 50%. No additional
cost for death animals was considered. Furthermore in Table 1 the number of piglets
weaned per reproductive cycle is also shown.

Daily feed intake of sows were taken from Kyriazakis and Whittemore (2006) and
updated weekly for each state proportionally to the duration of the corresponding state.
Feed cost is calculated weekly from feed intake and prices of feed. Price of feed as
parameter was obtained from the Annual Statistics of the Agricultural sector, 2009,
(http://www.mapa.es/es/estadistica/pags/anuario/introduccion.htm, accessed 16
February 2010) edited by the former Spanish Ministry of Agriculture, Food and
Fisheries. These are shown in Figure 2 where it is observed a clear increment in feeding cost of sows during the period 2007-2008 and part of 2009. Price of feed for piglets increased during 2007, following an irregular behaviour with steady periods and a hard decline by the end of 2008.

Unitary sale price parameters were extracted from the main auction market of pigs in Spain: MercoLleida (http://www.mercolleida.com, accessed 16 February 2010), settled in the same area where the farm was supposed to be operating. In particular, these data are registered and available in the Department of Agriculture from the autonomous government of Catalonia (http://www20.gencat.cat/portal/site/DAR/menuitem.3645c709047c363053b88e10b031e1a0/?vgnextoid=3fc4361d78b24110VgnVCM1000000b0c1e0aRCRD&vgnextchannel=3fc4361d78b24110VgnVCM1000000b0c1e0aRCRD&vgnextfmt=default, accessed 16 February 2010) in Spain. Historical prices since 1990 are recorded but only the last three available years were considered for this case, i.e. 2007, 2008 and 2009. For instance, Figure 3 shows the evolution week by week of the slaughterhouse price for culled sows (€/kg live weight) obtained from replaced sows. It is observed along time some seasonal pattern with lower prices in October and November while peaks from June to August are registered due to the increment in demand during summer time.

Figure 3 also describes the behaviour of the weekly price of piglets (€/piglet) regardless of their weight (i.e. standardised around 20 kg in Spain). Peaks and valleys in piglet price anticipate the evolution of slaughterhouse values that are a little bit delayed. In
2007 a noticeable decrease in price is observed leading to the lowest annual price after
summer time for the period considered (2007, 2008 and 2009).

For modelling purposes, purchase of gilts was calculated considering the price per kg as
that fixed by the slaughterhouse plus a 30% increment (Figure 3). Weights of sows per
cycle and reproductive state were borrowed from Kristensen and Søllested (2004). The
smooth variation in purchases from week to week, as well as minimum and maximum
bounds of gilts allowed to be purchased were set in $dz=5$, $lz=5$ and $uz=50$, respectively.

In addition to the figures described above, an average insemination cost per
insemination dose was fixed in 3€ and two doses were applied per mating.

3.2. Basic scenarios

It is well known the variability of several parameters involved in piglet production but
less the impact they have on herd dynamics over time and associated economic
performance. Past models revised by Plà (2007) focused on the analysis of sow farms at
the steady-state, i.e. under an infinite time horizon. However, the adjustments in herd
dynamics and derived productivity in the short term can be of more interest when it
comes to the transfer of results to the real world. For example, optimal management
strategies can be sub-optimal after changes in herd dynamics, environment or both.

Having this in mind and in agreement with general objectives of the work, two basic
scenarios were considered to perform an ex post analysis focusing on herd dynamics
and economic outcomes. The first one considered all parameters and values time-
dependent describing a more realistic environment. The second one, instead, considered
all parameters non time-dependent operating under an infinite planning horizon in a
similar way that optimization models do perform. This meant the same model was
solved with the only difference of taking average values for some parameters (prices
and litter size), making them constant over time. In Table 2 the parameters affected by
the scenario and corresponding mean and standard deviation values are shown.

**************Table 2*********************************

4. Computational results

4.1. Results and discussion of basic scenarios

The size of the model corresponding to the two basic cases resulted in 89,857 variables
and 81,638 constraints. The main result of interest was the number of new animals
entering weekly to the farm (purchased gilts), determined in agreement with the size of
facilities, replacement policy and reproductive management. Results in Figure 4 show
and compare the pattern of the purchase scheduling of new gilts over time (first year i.e.
52 weeks) in the two basic cases. Both are rather different however, the main noticeable
difference is that real values seem to lead to a sharper graph with more purchases in the
first basic case than when using average values in the second one. This is confirmed by
the replacement rate that was higher in the first basic case than in the second (63.79%
and 50.14% respectively) while herd size was almost stable for both situations (i.e.
average size of 2081 sows with a low coefficient of variation of 1%), but different herd
distributions over parities as shown by the average parity of sows at farrowing (3.30 and
3.58 respectively). In the second basic case, economic environment is stable given that
averaged prices are applied over time. Then it is possible to see better how purchases
grow before summer anticipating negative effects of the hot weather, decline in summer
and slightly recover again later. This effect is clearly perturbed in the first basic case by
the evolution of market prices over time. Thus, the use of real values in the first basic
case makes purchases to be more dynamic and to react face to economic environment. Economic results are also different (789.3 and 730.2 thousands of €/year) making sows of the first basic case more productive than those of the second basic case given the equal herd size. In other words, variability gives the opportunity of raising revenue. In fact, these results demonstrate firstly the adaptive variations of herd composition over time as response to changes in the environment and secondly the different herd structure derived from sets of parameters with equal mean values over the same time horizon.

The occupancy rate per farrowing facilities was in full accordance to the target of farrowings per week in both basic cases. This confirms the expected behaviour of the model, extracting the maximum profit of lactation facilities from where piglets (as main source of income in the farm) are produced.

Other effects on herd dynamics like summer season are also noticed in the rate of re-mating over total inseminations in agreement with previous higher levels of purchases just before the arrival of the summer season. Re-mating rate increases in summer up to a 14% just when the conception rate is the lowest. Otherwise the regular value of the re-mating rate out of the summer season varies between 8 and 10% while the averaged fertility is around 91%. This is in agreement with farmers practices who have to inseminate more sows in summer to maintain the maximum occupancy of farrowing facilitates. The replacement policy shows voluntary culling in all parities. Most sows are culled at the end of lactation (more than eighty percent) instead of after insemination. A higher
incidence of voluntary culling in the first basic case with respect to the second one would explain the difference between replacement rates mentioned above. This behaviour is also associated to extra benefits derived from market prices adopted in the first scenario.

4.2. Sensitivity analysis.

The basic scenarios were taken as reference for sensitivity analyses. Three different sets of parameters; conception rate (β_{t,c,r}), litter size (γ_{t,c}) and selling-purchasing prices (rp_t, rz_t,c, rx_t,c, ry_{t,c,g}, ch_{t,c,i}, cz_{t,c,k} and cz_{t,c}) were assessed. The variation applied is a plus/minus 20% of corresponding standard deviation (see Table 1 and 2) in each of the three sensitivity analysis performed by scenario (CR: fertility, LS: prolificity and PR: prices), so that, the term optimistic represents an increment (CR+, LS+ and Pr+) while the term pessimistic means a decrement (CR-, LS- and Pr-) in corresponding values of parameters. Summarized results for each sensitivity analysis are presented in Table 3. Some of them are equal or almost equal to the corresponding basic scenario (relative variations with respect to the corresponding basic case are shown in parenthesis, below the absolute value). Total profit is the only different outcome for each instance. It is observed how sensitivity in prices (Pr+ and Pr-) and litter size (LS+ and LS-), in this order, have a greater impact on economic results, more than this caused by variations in conception rate (CR+ and CR-). However, only variations in conception rates impacts on herd dynamics. In this sense, herd size, replacement rate and purchases are much more sensitive to CR variations than variations in Pr or LS. Now, comparing the same output variables from the different basic scenarios it is observed that relative variations (figures in parenthesis, Table 3) are always greater in the second basic case. However, the absolute values state otherwise. This similarity in trend, but not in relative impact may be interpreted as a higher, sometimes clearer, sensibility of simple models to show
reactions to permanent changes in parameters over time. The use of real variability of
parameters makes the system capable of smoothing this reaction in some way when
introducing the same kind of changes.

The sensitivity of LS and Pr in the second basic scenario does not affect herd dynamics
(i.e. herd size and replacement rate). Their impact is limited to the objective function
(i.e. economic outcome) because of the time independent nature of the parameters.
However, in the first basic case where real-based values are used, there is not such a
pure economic effect, but also a small influence on herd dynamics. This is induced by
the opportunity cost generated by the variation of market prices over time. Then, while
replacement rate has a 0% of variation in LS and Pr for the second basic scenario, this is
not so in the first basic scenario where relative variations of 0.03%, -0.12%, 0.17%, and
-0.29% were obtained in the replacement rate of LS+, LS-, Pr+ and Pr- respectively.
The latter results give a clue about a common practice of farmers where good
productivity level/prices lead to higher replacement rates, and bad productivity
level/prices lead to lower replacement rates and older herds. However, figures show that
the impact would be almost insignificant in contrast with real practices that are much
more exaggerated. This may suggest the need of suitable tools to value the impact of
current decisions on medium-term productivity derived from induced changes in herd
structure.
As mentioned, fertility always influences on herd structure and herd dynamics. Hence, positive or negative variations in conception rate makes replacement rate decrease or increase, even more if the productivity level (i.e. number of farrowings per week) has to be maintained. The impact on herd structure is obvious and shown in Table 4, where herd distribution for LS and Pr are not presented because they are technically similar to the corresponding basic cases (i.e. they have no practical impact on herd structure).

Clearly, it is observed how low conception rates lead to more sows in the first parity (i.e. higher replacement rate) and high conception rates lead to less sows in the first parity (i.e. lower replacement rate). This effect is even greater if herd size is considered given that low fertilities lead to bigger herd sizes for the same productivity level (Table 4). This is so because the number of inseminations and sows to be inseminated can be reduced according to a better conception rate and vice versa. Another effect to mention is the farmer workload that in the pessimistic case (CR-) is higher due to the increment of inseminations and re- inseminations to perform for attaining the same target of farrowings than those performed in the optimistic case (CR+).

The evolution over time of several indexes or variables gives additional insight. For instance, the target of farrowings was met all weeks, even those unaffected by the constraint during the first period, $T_1$. All instances showed a similar behaviour increasing herd size before summer to anticipate the decline in fertility and a reduction after the recovery of normal conception rates (Figure 5). In this way, purchase of gilts contributes to these adjustments in herd size but differently depending mainly on the basic scenario concerned which follows the same pattern displayed in Figure 4. Then for
the analyses based on the second basic scenario, where prices are constant over the whole time horizon, the pattern shows increments of purchases till summer and a decline later. The first basic scenario instead is affected by the variation on prices, and presents a more chaotic planning of purchases depending on the weekly prices. This behaviour is driven by the need to fully occupy lactation facilities and impacts on breeding facilities that act as a buffer of lactating sows according to expected fertility.

5 Conclusions

In this work a linear programming model for scheduling replacement and purchases in breeding farms producing piglets is formulated. The finite time horizon approach is shown to be suitable to explore and prepare the system to future variations in the short term including time varying parameters. Furthermore, the incorporation of a farrowing target quota allows the easier integration and coordination of piglet production into a pig supply chain. Two basic scenarios were proposed to assert the benefits of representing real variability against time homogeneity of parameters as infinite time horizon models do implicitly. As a result, a general similarity was found in trends and average outcomes, but not in the dynamics exhibited by both cases. The simplest basic scenario, the second one, was more sensitive to changes and easier to understand (e.g. scheduling of gilts’ purchases) whilst the use of real variability of parameters made the evolution over time of the first scenario case more difficult to interpret.

On the other hand, sensitive analyses demonstrated the different response of the herd when farm specific parameters are different. Changes in economic environment (selling prices and feeding cost or litter size performance) result in different objective function realisations, but very much less effect on herd dynamics. However, reproductive
performances impact on herd dynamics and bad ones represent increments of the herd size and number of inseminations (i.e. workload) and culled sows to maintain piglet production levels. Even though, some technical indexes like replacement rate may become confusing when comparing rates coming from different herd sizes and performances. Another model including more detailed constraints on economics is needed to explore in deep economic behaviour of the farm. This may suggest the lack of suitable tools to value and analyze the impact of current decisions on medium-term productivity as result of induced changes in herd structure. For instance, the common practice of modifying replacement policies as response to market crisis should be revised considering properly the subsequent impact on future productivity derived from induced changes in herd structure. Thus, finite time horizon models constitutes a valuable tool to understand adjustments of herd structure face to technical changes and to better support decisions in the short term.

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Appendix

The formulation of a model representing sow herd management through the reproduction and replacement management at farm level leads to consider the following decision variables and state variables that characterize the herd structure of the farm at any given time period:

Decision variables:

1. \( Z_{t,1} \) = number of purchased sows in the mating state, waiting for the first insemination of the cycle 1, at period \( t \),
2. \( UL_{t,c,l} \) = number of replaced sows at the end of the lactation state at period \( t \), cycle \( c \), at lactation week \( l \),
3. \( UZ_{t,c,r} \) = number of replaced sows at the end of the mating state at period \( t \), cycle \( c \), waiting for the insemination attempt \( r \),

State variables influenced by replacement and purchase decisions:

4. \( X_{t,c,g} \) = number of sows in gestation state at period \( t \), cycle \( c \), gestation week \( g \),
5. \( Y_{t,c,l} \) = number of sows in lactation state at period \( t \), cycle \( c \), lactation week \( l \),
6. \( Z_{t,c} \) = number of sows in the mating state, waiting for the first insemination of the cycle \( c \) (\( c > 1 \)), at period \( t \),
7. \( ZR_{t,c,r,k} \) = number of sows at the week \( k \) of the control state after the \( r^{th} \) insemination of the cycle \( c \), at period \( t \),
8. \( AB_{t,c} \) = number of sows with abortion at period \( t \), cycle \( c \),
9. \( D_{t,c,g} \) = number of dead sows at period \( t \), cycle \( c \), gestation week \( g \),

Hence, the proposed model maximizes the total profit of the production plan given by the objective function in (1), which represents the maximization of the total profit over
the finite time horizon, $T$, and reproductive cycle, $c \in C$. This function is subjected to a set of constraints (2)-(30).

Maximise $\Phi^{c} = \sum_{i \in T} \sum_{c \in C} r_{i,c}^{c} Y_{i,c}^{c} + \sum_{i \in T} \sum_{c \in C} \sum_{s \in S_{i}} \sum_{l \in L_{i}} r_{i,c}^{c} U_{i,c}^{c} + \sum_{r \in N_{r}} \sum_{c \in C} r_{r,c}^{c} U_{i,c}^{c} + r_{x,c}^{c} \cdot AB_{i,c}^{c}$

subject to:

5. $Z_{i,c} = z_{0}, \quad c \in C - \{l\}$

6. $Z_{R_{i,r,k}} = zr_{0_{i,r,k}}, \quad c \in C \quad r \in N_{r} \quad k \in S_{r}$

7. $X_{i,g} = x_{0_{i,g}}, \quad c \in C \quad g \in S_{g}$

8. $Y_{i,l} = y_{0_{i,l}}, \quad c \in C \quad l \in S_{l}$

9. $Z_{i,c}^{t} = Y_{i,c}^{t} - U_{i,c}^{t} \quad t \in T - \{l\} \quad c \in C - \{l\}$

10. $Z_{R_{i,r,l}} = Z_{i,l} \quad t \in T - \{l\} \quad c \in C$

11. $Z_{R_{i,r,l}} = (1 - \beta_{i,r,l}) \cdot Z_{R_{i,r,l}} - U_{i,c}^{t} \quad t \in T - \{l\} \quad c \in C \quad r \in N_{r} - \{l\}$

12. $Z_{R_{i,r,k}} = Z_{R_{i,r,k}} \quad t \in T - \{l\} \quad c \in C \quad r \in N_{r} \quad k \in S_{r} - \{l\}$

13. $X_{i,r} = \sum_{r \in N_{r}} \beta_{i,r,c} \cdot Z_{R_{i,r,c}} \quad t \in T - \{l\} \quad c \in C$

14. $X_{i,g} = \alpha_{i,r,g} \cdot X_{i,g} \quad t \in T - \{l\} \quad c \in C \quad g \in S_{g} - \{l\}$

15. $Y_{i,l} = \alpha_{i,l} \cdot X_{i,l} \quad t \in T - \{l\} \quad c \in C$

16. $Y_{i,l} = Y_{i,l}^{t} - U_{i,l}^{t} \quad t \in T - \{l\} \quad c \in C \quad l \in S_{l} - \{l\}$

17. $\sum_{c \in C} \sum_{c \in C} \sum_{c \in C} Z_{R_{i,r,k}} \leq cb \quad t \in T$

18. $\sum_{c \in C} \sum_{c \in C} \sum_{c \in C} X_{i,g} \leq cp \quad t \in T$
\[
\sum_{c \in C} X_{t,c,g}^e + \sum_{c \in C} \sum_{l \in Sl} Y_{t,c,l} \leq \epsilon f \quad t \in T
\] (16)

\[
|Z_{t,c,l} - Z_{t,i,c}| \leq dz \quad t \in T - \{t^*\}
\] (17)

\[
iz \leq Z_{t,i} \leq uz \quad t \in T
\] (18)

\[
\sum_{c \in C} Z_{t,c,l} \geq zf
\] (19)

\[
\sum_{c \in C} \sum_{l \in Sl} \sum_{r \in Nr} ZR_{t,c,r,k} \geq zrf \quad r \in Nr
\] (20)

\[
\sum_{c \in C} \sum_{l \in Sl} Y_{t,c,l} \geq xf \quad g \in Sg^r
\] (21)

\[
\sum_{c \in C} \sum_{l \in Sl} \sum_{l \in Sl} Y_{t,c,l} \geq yf_i \quad l \in Sl
\] (22)

\[
UZ_{t,c,l} = (1 - \beta_{t,c,l}^e) \cdot ZR_{t,c,r,k} \quad c \in C \quad t \in T
\] (23)

\[
UL_{t,c,l} = Y_{t,c,l} \quad t \in T - \{t^*\}
\] (24)

\[
UZ_{T,c,l} \leq ZR_{T,c,r,k} \quad c \in C \quad r \in Nr - \{r^*\}
\] (25)

\[
UL_{T,c,l} \leq Y_{T,c,l} \quad c \in C \quad l \in Sl
\] (26)

\[
\alpha_{t,c,00} X_{t,c,10} = AB_{t,c} \quad c \in C \quad t \in T
\] (27)

\[
\alpha_{t,c,00}^r X_{t,c,10} = D_{t,g} \quad c \in C \quad t \in T
\] (28)

\[
\sum_{c \in C} \sum_{l \in Sl} UL_{t,c,l} + \sum_{c \in C} \sum_{r \in Nr} UZ_{t,c,r,k} + \sum_{c \in C} AB_{t,c} + \sum_{g \in G} D_{t,g} \geq \varphi \quad t \in T
\] (29)

\[
\sum_{c \in C} Y_{t,c,l} \geq fq \quad t \in T - T_i
\] (30)

where:

\[
z\theta_c = \text{initial stock of animals in mating state at cycle } c,
\]
$z_{r0_{c,r,k}}$ = initial stock of animals in control state at cycle $c$, waiting for the insemination attempt $r$ and week $k$,

$x_{0_{c,g}}$ = initial stock of animals in gestation state at cycle $c$ and gestation week $g$, and

$y_{0_{c,l}}$ = initial stock of animals in lactation state at cycle $c$ and lactation week $l$,

$\alpha_{t,c,g}$ = survival rate of gestation at period $t$, cycle $c$ and gestation week $g$,

$\beta_{t,c,r}$ = fertility rate of mating at period $t$, cycle $c$ and waiting for the mating attempt $r$,

$k^*$, $l^*$, $g^*$ = the last elements of the corresponding states sets $Sr$, $Sl$, $Sg'$ respectively.

$zf$ = the minimum number of animals in mating state at the end of the planning horizon,

$z_{rfr,k}$ = the minimum number of animals in control state at the end of the planning horizon waiting for the insemination attempt $r$ and week $k$,

$xf_g$ = the minimum number of animals in gestation state at the end of the planning horizon at gestation week $g$,

$y_{fl}$ = the minimum number of animals in lactation state at the end of the planning horizon at lactation week $l$,

$\alpha_{r,t,c,10}$ the abortion rate per cycle in week 10$^{th}$ of gestation at period $t$.

$\alpha_{r',t,c,g}$ the mortality rate per cycle in week $g$ of gestation at period $t$. 


Table 1. Main fertility rates, $\beta(t,c,r)$, of mating state at $t$ time period, $c$ cycle, $r$ insemination attempt and number of weaned piglets per reproductive cycle, $\gamma_{tc}$.

<table>
<thead>
<tr>
<th>Cycle ($c$)</th>
<th>Mating ($r$)</th>
<th>Weaned piglets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0.890</td>
<td>0.863</td>
</tr>
<tr>
<td>2</td>
<td>0.924</td>
<td>0.896</td>
</tr>
<tr>
<td>3</td>
<td>0.931</td>
<td>0.903</td>
</tr>
<tr>
<td>4</td>
<td>0.934</td>
<td>0.896</td>
</tr>
<tr>
<td>5</td>
<td>0.922</td>
<td>0.884</td>
</tr>
<tr>
<td>6</td>
<td>0.912</td>
<td>0.885</td>
</tr>
<tr>
<td>7</td>
<td>0.906</td>
<td>0.878</td>
</tr>
<tr>
<td>8+</td>
<td>0.903</td>
<td>0.875</td>
</tr>
</tbody>
</table>

* Data extracted from an anonymous farm belonging the BDporc databanc with 2000 sows.
Table 2. Mean and SD of parameters involved in the second basic case.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Units</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_l$</td>
<td>Value of weaned piglet</td>
<td>€/piglet</td>
<td>27.14744</td>
<td>8.04327</td>
</tr>
<tr>
<td></td>
<td>Pre-starter</td>
<td>€/kg</td>
<td>0.35693</td>
<td>0.03897</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Slaugtherhouse price</td>
<td>€/kg</td>
<td>1.09403</td>
<td>0.11969</td>
</tr>
<tr>
<td>$C_g$</td>
<td>Gilt purchase</td>
<td>€/kg</td>
<td>1.09403</td>
<td>0.11969</td>
</tr>
<tr>
<td>$R_u$</td>
<td>Slaugtherhouse price</td>
<td>€/kg</td>
<td>1.09403</td>
<td>0.11969</td>
</tr>
<tr>
<td>$R_z$</td>
<td>Feed</td>
<td>€/kg</td>
<td>0.22756</td>
<td>0.03145</td>
</tr>
<tr>
<td>$R_{rz}$</td>
<td>Feed</td>
<td>€/kg</td>
<td>0.22756</td>
<td>0.03145</td>
</tr>
<tr>
<td>$R_x$</td>
<td>Feed</td>
<td>€/kg</td>
<td>0.22756</td>
<td>0.03145</td>
</tr>
<tr>
<td>$R_y$</td>
<td>Feed</td>
<td>€/kg</td>
<td>0.22756</td>
<td>0.03145</td>
</tr>
<tr>
<td>$Gama$</td>
<td>Litter size</td>
<td>#</td>
<td>11.42500</td>
<td>0.89861</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Piglet mortality</td>
<td>%</td>
<td>0.11833</td>
<td>0.00577</td>
</tr>
</tbody>
</table>
Table 3. Sensitivity analysis over several parameters (CR: Conception Rate; LS: Litter Size; Pr: real prices based on time series from 2007 to 2009). Optimistic and pessimistic represents respectively a plus-minus 20% of standard deviation of involved parameters.

<table>
<thead>
<tr>
<th></th>
<th>Optimistic</th>
<th></th>
<th>Pessimistic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>€</td>
<td>HS</td>
<td>%Rep</td>
<td>€</td>
</tr>
<tr>
<td>CR</td>
<td>792.45</td>
<td>2162</td>
<td>65.15%</td>
<td>783.70</td>
</tr>
<tr>
<td></td>
<td>(0.40%)</td>
<td>-(0.86%)</td>
<td>(2.13%)</td>
<td>-(0.71%)</td>
</tr>
<tr>
<td>LS</td>
<td>849.31</td>
<td>2181</td>
<td>63.81%</td>
<td>729.36</td>
</tr>
<tr>
<td></td>
<td>(7.60%)</td>
<td>(0.00%)</td>
<td>(0.03%)</td>
<td>-(7.60%)</td>
</tr>
<tr>
<td>Pr</td>
<td>859.07</td>
<td>2181</td>
<td>63.90%</td>
<td>719.61</td>
</tr>
<tr>
<td></td>
<td>(8.83%)</td>
<td>(0.00%)</td>
<td>(0.17%)</td>
<td>-(8.83%)</td>
</tr>
<tr>
<td>CR</td>
<td>733.90</td>
<td>2161</td>
<td>50.69%</td>
<td>724.64</td>
</tr>
<tr>
<td></td>
<td>(0.50%)</td>
<td>-(0.91%)</td>
<td>(1.07%)</td>
<td>-(0.77%)</td>
</tr>
<tr>
<td>LS</td>
<td>790.20</td>
<td>2180</td>
<td>50.16%</td>
<td>670.30</td>
</tr>
<tr>
<td></td>
<td>(8.21%)</td>
<td>(0.00%)</td>
<td>(0.00%)</td>
<td>-(8.21%)</td>
</tr>
<tr>
<td>Pr</td>
<td>796.06</td>
<td>2180</td>
<td>50.16%</td>
<td>664.44</td>
</tr>
<tr>
<td></td>
<td>(9.01%)</td>
<td>(0.00%)</td>
<td>(0.00%)</td>
<td>-(9.01%)</td>
</tr>
</tbody>
</table>

(In parenthesis relative variation with respect corresponding basic case)
Table 4. Herd distribution over parities. Sensitivity analysis on conception rate (CR-: basic values minus 20% of SD; CR+: basic values plus a 20% of SD) is compared with respective basic cases (first and second).

<table>
<thead>
<tr>
<th>Parity</th>
<th>First Basic Case</th>
<th></th>
<th></th>
<th>Second Basic Case</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basic</td>
<td>CR-</td>
<td>CR+</td>
<td>Basic</td>
<td>CR-</td>
<td>CR+</td>
</tr>
<tr>
<td>1</td>
<td>25.08%</td>
<td>25.15%</td>
<td>24.93%</td>
<td>20.22%</td>
<td>20.55%</td>
<td>19.90%</td>
</tr>
<tr>
<td>2</td>
<td>19.33%</td>
<td>19.53%</td>
<td>19.03%</td>
<td>18.58%</td>
<td>18.80%</td>
<td>18.33%</td>
</tr>
<tr>
<td>3</td>
<td>15.23%</td>
<td>15.26%</td>
<td>15.31%</td>
<td>15.86%</td>
<td>15.80%</td>
<td>15.92%</td>
</tr>
<tr>
<td>4</td>
<td>12.79%</td>
<td>12.71%</td>
<td>12.89%</td>
<td>13.56%</td>
<td>13.46%</td>
<td>13.69%</td>
</tr>
<tr>
<td>5</td>
<td>10.43%</td>
<td>10.37%</td>
<td>10.47%</td>
<td>11.25%</td>
<td>11.13%</td>
<td>11.37%</td>
</tr>
<tr>
<td>6</td>
<td>7.87%</td>
<td>7.83%</td>
<td>8.01%</td>
<td>8.73%</td>
<td>8.63%</td>
<td>8.84%</td>
</tr>
<tr>
<td>7</td>
<td>5.44%</td>
<td>5.35%</td>
<td>5.47%</td>
<td>6.33%</td>
<td>6.29%</td>
<td>6.37%</td>
</tr>
<tr>
<td>8+</td>
<td>3.83%</td>
<td>3.81%</td>
<td>3.89%</td>
<td>5.48%</td>
<td>5.34%</td>
<td>5.59%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parity</th>
<th>First Basic Case</th>
<th></th>
<th></th>
<th>Second Basic Case</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basic</td>
<td>CR-</td>
<td>CR+</td>
<td>Basic</td>
<td>CR-</td>
<td>CR+</td>
</tr>
<tr>
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<td>20.22%</td>
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</tr>
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</tr>
<tr>
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<td>3.81%</td>
<td>3.89%</td>
<td>5.48%</td>
<td>5.34%</td>
<td>5.59%</td>
</tr>
</tbody>
</table>
Figure 1. Sow herd dynamic behaviour.

Figure 2. Weekly price of feed (€/kg) for sows and piglets. Period 2007-2009.
Figure 3. Weekly price of sows (€/kg live weight) and piglet weaned (€/piglet). Period 2007-2009.

Figure 4. Scheduling of purchases for the 1st and 2nd basic case
Figure 5. Herd size over time for particular sensitivity analysis