Tittle: Comparison of different nonlinear functions and statistical models to describe beef cattle growth.

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Short title: Growth curves in beef cattle

Abstract.

The aim of this study was to describe the growth of Parda de Montaña females. Firstly, five non-linear functions (Richards, Brody, Von Bertalanffy, Gompertz and Logistic) were used to fit bodyweight (BW)-age data using the *nlme* procedure of R statistical software. Comparisons were made among these functions for goodness of fit, standardized residuals and biological interpretability of the growth curve parameters. The Richards function showed the best goodness of fit. Both the Richards and Von Bertalanffy functions estimated more accurately BW at different time periods (at birth, during suckling, during rearing until the first mating and until the first calving) and underestimated or overestimated the BW to a greater or lesser extent, according to the standardized residuals. The Brody function had the best estimation of weight at maturity (599 kg) and the Richards function gave the closest estimation of birth weight (41 kg), average absolute growth rate (0.845 kg/day), age at puberty (56% of asymptotic BW; 365 days) and mature age (98% of asymptotic weight; 1260 days). Different residual and random modeling structures were compared using the Richards function. The best goodness of fit was obtained in the model which included the constant plus power variance function and parameters *A*, *k* and *n* as random effects. Finally, the predictive

ability of this model was checked. A strong correlation between the predicted and actual adult weight of cows with incomplete data (R=0.85) was observed.

Introduction.

In animal science, growth is usually defined as an increase in tissue mass caused by hyperplasia early in life and hypertrophy later in life (Owens *et al.* 1995). The accurate knowledge of the growth curve is important due to its relation with the efficiency of production (Fitzhugh, 1976). The analysis of growth curves was initiated in the physical sciences, more fully developed in the biological sciences, and used in studies of the size and health of plants, animals and humans (McArdle, 2001).

Mathematical models can be used to describe animal growth using the bodyweights recorded throughout the animal's life. Since the publication of Brody (1945), several theories and sophisticated algorithms for analysis have been developed. Fitzhugh (1976) discussed and appraised different techniques for fitting and analyzing growth curves and their biological interpretation.

In beef cattle, different equations to model growth have been used and compared in Hereford (Brown *et al.* 1976), Angus (Beltran *et al.* 1992), Retinta (López de Torre *et al.* 1992), Belgian Blue (de Behr *et al.* 2001), Salers (Garcia et al., 2008) and Nelore (Forni *et al.* 2009) females, among others. Neither the best equation to describe growth nor the way to model the residual variance are clear, they depend on the breed (Freetly *et al.* 2011) and sometimes on the structure of the data (Forni *et al.* (2009).

The aim of this study was to describe the growth of Parda de Montaña females from birth to adult age. Firstly, various nonlinear functions were compared to choose the best one to describe the growth according to the goodness to fit. Then, different residual and random modeling structures were compared using the best function previously selected. Finally, the predictive ability of the best nonlinear mixed effects model was checked.

Materials and methods.

Data were collected in La Garcipollera Research Station (Spain, 42° 37′ N; 0° 30′ W; 945 m above sea level), in the mountain area of the southern Pyrenees (Spain), and in CITA research centre in Zaragoza (41° 43' N; 0° 48' W; 225 m above sea level), for 23 years, between 1987 and 2010, from a Parda de Montaña cattle herd (60-100 adult cows over the study period).

Parda de Montaña (PM) is a suckler cattle breed widely spread throughout northern Spain that came from the ancient Brown Swiss and its crosses with local breeds. It had been used as a dual-purpose breed, milk-beef (Álvarez-Rodríguez *et al.* 2009), but in the last decades it has been selected basically for beef production and mothering abilities (weaning weight and calving ease).

The management of this herd of Parda de Montaña cattle consisted of housing during winter, grazing on high mountain pastures (1500-2200 m above sea level) during the summer and grazing on valley meadows and forest pastures (945-1500 m above sea level) during spring and autumn. Concerning the reproductive management, there are two calving seasons, spring (March to May) and autumn (September to November). Further details can be found in (Casasus *et al.* 2002). Cows remained in the herd until natural death occurred or they were culled for either sanitary reasons or reproductive failure.

The database was constructed with a total of 24174 bodyweights (BWs) registered from 1697 females. They were weighed on the first week of life, at weaning, during rearing, at calving, at the end of lactation, when they were turned-out to high mountain pastures and turned-into the farm facilities, and also at different frequencies in between these time points. Although all animals did not have the complete set of weights throughout their life, the data set had enough number of records on each age (Fig. 1). Within an age (in weeks), records outside the range of 3 standard deviations from the observed bodyweight (BW) were considered as outliers and were not included in the analyses.

(Insert Fig. 1 here)

Five nonlinear functions frequently used for the description of growth curves in cattle were studied: Richards, Brody, Von Bertalanffy, Gompertz and Logistic as proposed by Fitzhugh (1976). These nonlinear functions are detailed in the Table 1. The functions were compared considering the goodness of fit and standardized residuals. Goodness of fit of the different models was evaluated using maximum likelihood of the model (the higher the better), Akaike information criterion (AIC) (the lower the better) and Bayesian information criterion (BIC) (the lower the better). A Likelihood Ratio Test (LRT) was performed to compare the model that had the best the goodness of fit with the remaining models. The models were fitted using the *nlme* procedure of the version 2.13.2 of R statistical software (Pinheiro & Bates 2000). The model comparison was performed with *anova* procedure of R statistical software. Standardized residuals of the different functions were obtained as the difference between estimated BW and actual BW divided by the standard deviation of the actual BW at the age of estimation.

The biological interpretation of the growth curve parameters (Table 1) is as follows:

- y_t = observed BW at age t.

- A = the asymptotic limit of the BW when age t approaches to infinity, interpreted as average size at maturity independent of short-term fluctuation of size in response to environmental effects such as the climate, the type and availability of feed, or due to the physiological status.

- *b* = integration constant, a time scale parameter.
- *k* = ratio of the relative intensity of growth that estimates the maturation rate of the curve. In order to obtain a proper convergence parameter k was scaled to obtain estimates of random parameters variances with similar magnitude.
- *n* = shape parameter determining the position of the inflection point (POI) of the curve.

Further parameters were obtained from the abovementioned ones: birth weight (calculated at $t = 0$), BW (y^+) and age (t+) at the point of inflection. Useful growth parameters can be derived from the method such as: average absolute maturing rate (v) (kg/d) and average absolute growth rate (v^+) (kg/d at POI), age at puberty (estimated at 56% of mature BW, according to Freetly *et al*. (2011), $t^{56\%}$) and age to reach maturity (estimated at 98% of asymptotic BW, according to Johnson *et al.* (1990), $t^{98\%}$).

(Insert Table 1 here)

The residual variance was included in the mixed models with no heteroscedasticity (constant residual variance; M1), using the power variance function (variance related with BW; M2) or the constant plus power variance function (equivalent to power variance function but with a constant value at low BW; M3). Mixed models were used to adjust the functions to data with parameters of the function defined as fixed effects (i.e. describing the population) and as random effects (i.e. describing the individual deviation from the mean value of the population). The function with best goodness of fit was selected for further statistical analyses. In the selected function, different random effects were also tested: all parameters (*A*, *k*, *n* and *b*), parameters *A*, *k* and *n*, parameters *A* and *k*, parameter *A* and *n* and only the parameters *A*. The models were compared considering the lack of solution (no convergence in the iterative procedure) and the goodness of fit, as described above. Growth curve parameters obtained for each model were compared using a t test.

Finally, a subset of our data was constructed including only cows with BWs available during their lifespan (complete dataset) to study the predictive ability of the best nonlinear mixed effects model. Using the best model, the "actual" adult deviation of each cow from the random effect *A* was estimated. The complete dataset was used to construct a second subset (reduced dataset). In the reduced dataset, half of the cows had BW over their whole lifespan and the other half only had BWs before 200 days of age, because BWs after this age were deleted. Then, the "predicted" adult deviation of all cows with reduced dataset was obtained using the best function adjustment and the "actual" and "predicted" adult weight of cows were compared.

Results and discussion.

The Richards function had the best goodness to fit because it had the maximum likelihood and the lowest AIC and BIC values (Table 2) and so, the other functions were compared with it. High LRT values were obtained for all comparisons showing that the Richards function had a better adjustment than the others ($P < 0.001$). The Von Bertalanffy function had a good adjustment whereas the Logistic function had the worst adjustment. The Richards and Brody functions were selected among others because they had the best goodness to fit according to the sum of squared deviations (Brown *et al.* 1976; DeNise and Brinks 1985; Doren *et al.* 1989). These two equations were chosen by Beltran *et al.* (1992) to describe the growth of Angus cattle because of their goodness of fit and because the parameter estimates can be interpreted easily. The Brody function represented more accurately Nelore cattle growth whereas the Logistic function was the least accurate according to AIC and BIC (Forni et al. 2009).

(Insert Table 2 here)

The observed BW and mean predicted curve obtained with the different functions are shown in Fig. 2. The accuracy of the prediction varied over the different time periods: at birth, during suckling (until 140 to 170 days old, depending on the calving season), during rearing from weaning until the first mating (until 600 to 660 days old), then from the first mating until the first calving (910 to 1000 days old) and finally during maturity. According to the standardized residuals (Fig. 3), the Richards and Von Bertalanffy functions estimated more accurately the BW at all ages. Similarly, Brown *et al.* (1976) reported that the Richards function gave a generally unbiased fit at all ages and Von Bertalanffy function fit reasonably well over all ages, although it usually overestimated BWs at ages prior to 6 months in Hereford and Jersey cattle.

(Insert Fig. 2 here)

Concerning the remaining functions studied in the current analysis, the Logistic and Gompertz functions tended to overestimate birth weights. The Brody function overestimated the BW during the suckling period whereas the Gompertz and Logistic underestimated it. From 360 days to the first mating, the behaviour of these three functions was inverse, the Logistic and Gompertz functions overestimated the BW and the Brody function underestimated it. During maturity, all functions underestimated or overestimated the BW to a greater or lesser extent, alternatively. The results agree partially with those reported by Brown *et al.* (1976). They reported that the Brody, Gompertz and Logistic functions over- and underestimated BW in different periods but not in the same way as these functions in the current analysis. In the abovementioned study, the Gompertz function consistently overstimated early BWs, the Brody function tended to either over or underestimate BWs prior to 6 months but fitted the observed data well for BWs after 6 months whereas the Logistic function tended to overestimate early BWs and underestimate the mature BW. In Nelore cattle, the Brody function provided more accurate estimates of birth weights whereas the Logistic function overestimated the birth weights (Forni *et al.* 2009). The differences between studies could be related to the different maturing rate of the breeds studied, and to the effect of the feeding management on the shape of the growth curve.

(Insert Fig. 3 here)

Within the function that better fitted the data, Forni *et al.* (2009) found that the poor fitting until 48 months of age was explained by a wide relative variation of records in early life. In our analyses, the greater variability in fit was found after first calving (30-months-old). In this period, BW of cows could vary: i) due to the physiological state, a dry cow can be 10% lighter than the same cow being 9 months pregnant (INRA, 1978), and ii) as a response to grazing, in the management scenario where data were obtained, a cow could gain 12% of BW in the summer grazing period (Casasus *et al.* 2002).

The parameters estimated from the different nonlinear functions are given in Table 3. The letters used to represent those parameters are the same in all functions but they do not have the same mathematical meaning and their comparison among functions is not possible, except for parameter *A*, which represents the asymptotic BW. According to the different models, mature BW ranged from 489 to 599 kg, for the Logistic and Brody functions, respectively. Bodyweight of multiparous Parda de Montaña cows at calving was 574 and 599 kg in the spring and autumn calving seasons, respectively (Casasus *et al.* 2002). Parameter *A* estimated with the Brody function was the closest to the abovementioned values, followed by the Richards function, which underestimated the adult BW. Conversely, better estimates of mature BW were obtained with the Richards function than with the Brody function in Angus cattle (Beltran *et al.* 1992) or with the Von Bertalanffy function than with the Brody function in Retinta cattle (López de Torre *et al.* 1992). The Von Bertalanffy function was also chosen to define the adult BW of different strains of Holstein-Friesian cattle (Berry *et al.* 2005). In the current analysis, the least accurate estimate was given by the Logistic function as it has been reported in Nelore (Forni *et al.* 2009) and in Holstein and Ayrshire cattle (Perotto *et al.* 1992).

(Insert Table 3 here)

The mean predicted curve obtained by the Logistic function was below the observed values (Fig. 2) and had underestimated mature weight (Table 3). This function made biased estimates of the random effects for the cows depending on the structure of the available data (Fig. 4). It provided a biased estimation of the *A* random parameter depending on the number of records available per cow whereas the Richards function estimations were relatively unbiased. The Logistic function tended to underestimate *A* for animals with a low number of records (i.e. with actual data until weaning) and overestimate *A* for animals with a high number of records (i.e. with data from birth to mature age).

(Insert Fig. 4 here)

The calculated birth weight ranged between 38 kg in the Brody function to 59 kg in the Logistic function (Table 4). The birth weight of Parda de Montaña female calves was 40.2 to 41.9 kg (Villalba *et al.* 2000; Casasus *et al.* 2002), which is close to the birth weight calculated with the Richards function.

All the functions, but the Brody function, are designed to include a point of inflection (POI) when growth rate changes from an increasing to a decreasing function of age. The Von Bertalanffy, Gompertz and Logistic functions have a fixed POI at some proportion of the mature size (8/27, e^{-1}, and 0.5, respectively) and so BW (y^+) and age at the *POI* (t^+) only depend on estimated mature BW. Consequently, these parameters (Table 4) had relatively low coefficients of variation (2.0 to 2.5%). Only the Richards function allows for a variable POI derived from the parameter *n* but the scarcity of observations in the segment of the curve around the inflection point may lead to inappropriate conclusions, because the curve shape seems to be the aspect of growth that is most sensitive to environmental factors (López *et al.* 2000). In the current analysis, there were enough data at all ages, however, and the estimated Richards function had the POI at 21% of mature size and showed the lowest y^+ and t^+ values with a coefficient of variation of approximately 7%. Other authors describe the growth in only one phase (Beltran *et al.* 1992) because *n* values obtained were not within the range that allows the calculation of the POI.

The parameters *v* and v^+ calculated with the Logistic function (Table 4) were biologically unlikely when they were compared with growth rates reported in previous studies. Growth rates at 120, 150 and 180 days in Parda de Montaña females were 0.831, 0.828 and 0.826 kg/day (Villalba *et al.* 2000), respectively, which is close to v^+ value calculated with the Richards function. Casasus *et al.* (1995) reported weight gains of 0.830 kg/day in grazing 15-month-old heifers. Variation of the growth parameters calculated with the Richards function were lower, approximately 2.5%, than the variation of these parameters calculated with the Gompertz and Von Bertalanffy functions, approximately 5.0%. With the Brody function, only parameter *v* could be calculated and it was similar to the one estimated with the Richards function.

Within *Bos taurus*, the relative range in proportion of mature BW at puberty (56-58%) is highly conserved, suggesting that proportion of mature BW is a more robust predictor of age at puberty across breeds than absolute weight or age (Freetly *et al.* 2011). Using this proportion, the age at puberty estimated with the different nonlinear functions ($t^{56\%}$ in Table 4) varied between 280 and 420 days calculated with the Logistic and Brody functions, respectively. Revilla *et al.* (1992) reported that Parda de Montaña heifers reached puberty at 378 days and 329 kg, being both close to those calculated with the Richards function (Table 4). Different studies reported that Brown Swiss reached puberty at 344 days and 297 kg (Gregory *et al.* 1978), 349 days and 281 kg (Laster *et al.* 1979) and 317 days and 305 kg (Ferrell, 1982). The difference in POI observed between Parda de Montaña and Brown Swiss (despite their common origin) could be explained because breeds that have been selected for milk production (Brown Swiss) reach puberty at a younger age and at a lighter weight, relative to mature weight, than breeds selected solely for beef production (Ferrell, 1982), as Parda de Montaña in the last decades.

Johnson *et al.* (1990) referring to previous studies of Brody and Taylor, considered animals to be fully mature when they had attained 98% of their asymptotic weight ($t^{98\%}$ in Table 4). Large differences were observed between the functions, clearly the Logistic and Gompertz functions had considerably underestimated age at maturity (2.0 and 2.5-years-old, respectively) and the Brody overestimated it (5.7-years-old). As the Richards function has been the most accurate and has shown the best fit, the estimation of the adult age could be done using this function, so the age to reach maturity of Parda de Montaña cows could be set at about 4.0-years-old.

(Insert Table 4 here)

The following analyses compared different types of modeling random and residual effects using the Richards function because it was chosen as the best function according to its better goodness of fit and the behavior of residuals. The best model was the M3 because it had the maximum likelihood and the lowest AIC and BIC values (Table 5). M3 model included a heterogeneous constant plus power variance based in two parameters, one for BW close to 0 and the second one as a power of the absolute value of BW. High LTR values were obtained for all comparisons indicating that model M3 had a better adjustment than the other models ($P < 0.001$), especially in comparison with the model with homogeneous (M1) variance which was the worst option. Pinheiro and Bates (2000) considered that the model M3 was better than the model with only power variance (M2) because it generally gives a more realistic output when the variance covariate was close or equal to 0.

(Insert Table 5 here)

Regarding the inclusion of random effects, the model did not converge in the iterative procedure when all parameters were included (*A*, *k*, *n* and *b*). A possible reason could be that there was a strong correlation among these parameters in our fittings (either in random but also in the fixed component $R > 0.9$) as described in other studies (Fang and Bailey 2001; Giraldo *et al.* 2002). Some studies even used a modified Richards function with only three parameters (Nahashon *et al.* 2006). In our analyses, the model which included the parameters *A*, *k* and *n* as random effects, showed the maximum likelihood, but the estimated standard deviation for the *k* random effect was very low and the correlation between *k* and *n* random effects was close to 1. This suggests that one of these parameters can be treated as a fixed effect. Pinheiro and Bates (2000) stated that creating a better-fitting model for the fixed effects, by including their dependence on covariates, reduced the need for random-effects terms. In these cases, the between-group parameter variation is mostly explained by the covariates included in the model. Accordingly, in our analyses the models M4 and M5 (with only one of the two random effects, either *n* or *k*) showed lower AIC and BIC than model M3 and so they can be considered more parsimonious because they included fewer random effects.

The estimated fixed parameters and their standard errors using the different models with the Richards function are shown in Table 6. All the estimated parameters of the model M1 were different from those obtained with the models that included the heterogeneity of residuals ($P < 0.05$). In the simplified model (M1), *A* was overestimated while *k* was underestimated compared to those of the models that had better adjustment to the observed data (M2-M6). Within models that include constant plus power variance structure, the exclusion of some random effect in models M4 and M6 led to significant differences ($P < 0.05$) for *b*, *k*, and *n* fixed parameter estimates.

(Insert Table 6 here)

Concerning the predictive ability of the mixed models using M3 with the Richards function, there was a strong correlation between the “predicted” and “actual” adult BW of cows included in the reduced dataset (R=0.85; Fig. 5). It even predicted reasonably well the mature BW of cows with only three weights before weaning. Bullock *et al.* (1993) reported a low phenotypic correlation between mature and birth or weaning BW (0.33 and 0.32 respectively), but a medium to high genetic correlation (0.64 and 0.80 for birth and weaning BW, respectively). The inclusion of random effects in the nonlinear models (even when genetic relationships between animals were not included) allowed an approximation to the mature BW based on two sources of information, first the actual data at young ages and second the pattern of growth obtained from animals with data within all ages. This implies that in experiments of characterization of growth curves and mature BW an optimal protocol can be designed to avoid the control of all the cows at all ages.

(Insert Fig. 5 here)

Conclusion.

The Richards function is the best function to describe growth of Parda de Montaña females. The use of nonlinear mixed models allowed the description not only of the mean curve but also the individual curve from birth to maturity for each female even when dataset of an animal was not complete. Obtaining the function parameters with biological meaning for each individual will be very useful to discuss the efficiency at an animal level.

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Table 1. Nonlinear functions applied to describe the growth curves.

y_t = observed BW at age *t*; *A* = the asymptotic limit of the BW when age *t* approaches to infinity; *b* = integration constant; *k* = ratio of the relative intensity of growth; *n* = shape parameter determining the position of the inflection point (POI) of the curve.

Name	Functions
Richards	$y_t = A (1 \pm be^{-kt})^{-1/n}$
Brody	$y_t = A (1 - be^{-kt})$
Von Bertanlaffy	$y_t = A (1 - be^{-kt})^3$
Gompertz	$y_t = A \exp(-be^{-kt})$
Logistic	$y_t = A (1 + be^{-kt})^{-1}$

Table 2. Maximum likelihood, Akaike information criterion (AIC) and Bayesian information criterion (BIC) of the different nonlinear functions and comparison with Richards function using *anova* and Likelihood Ratio test (LRT). P-value and LRT calculated calculated with respect to the best (Richards) function.

Functions Criteria	Richards	Brody	Von Bertalanffy	Gompertz	Logistic
Likelihood	-113682	-115415	-114429	-116324	-120344
AIC	227398	230854	228883	232670	240713
BIC	227536	230951	228980	232759	240810
P-value		<0.0001	<0.0001	<0.0001	<0.0001
LRT		3466	1495	5284	13325

Table 3. Statistics of the growth curve estimated parameters.

A = the asymptotic limit of the BW when age *t* approaches to infinity; *b* = integration constant; *k* = ratio of the relative intensity of growth; *n* = shape parameter determining the position of the inflection point (POI) of the curve.

Functions	Richards		Brody		Von Bertalanffy		Gompertz		Logistic	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
Parameter										
A	568	2.5	599	2.1	546	2.2	520	2.3	489	2.8
b	-0.798	0.0041	0.936	0.0003	0.569	0.0005	2.427	0.0033	7.349	0.0329
k (x1000)	2.713	0.0209	1.805	0.0069	3.531	0.0262	4.704	0.0379	8.011	0.0780
n	-0.611	0.0075								

Table 4. Statistics of biologically traits inferred from the growth curves.

y^+: BW and age (t^+*: age*, at the point of inflection (POI); *v:* Average absolute maturing rate; v^+: average absolute growth rate at POI; $t^{56\%}$*: a*ge at puberty; and $t^{98\%}$: time to reach maturity.

Functions	Richards	Brody	Von Bertanlaffy	Gompertz	Logistic
Parameter					
Birth weight (kg)	41	38	44	46	59
y^+ (kg)	121		162	191	244
t^+ (days)	98		152	189	249
v (kg/day)	0.555	0.541	0.578	0.612	0.652
v^+(kg/day)	0.845		0.857	0.900	0.979
$t^{56\%}$ (days)	365	420	330	305	280
$t^{95\%}$ (days)	1260	1620	1000	820	620

Table 5. Maximum likelihood, Akaike information criterion (AIC) and Bayesian information criterion (BIC) of different random effect components and different variance structure included in the Richards function model. P-value and LRT calculated calculated with respect to the best (M3) function.

Models [A]	M1	M2	M3	M4	M5	M6
Variance structure	Constant	Power	Constant plus Power			
Random parameters		A k n		A k	A n	A
Criteria						
Likelihood	-119131	-113752	-113682	-114659	-113752	-114659
AIC	238292	227536	227398	229347	227531	229343
BIC	238413	227665	227536	229460	227636	229440
P-value	<0.001	<0.001		<0.001	<0.001	<0.001
LRT	10898	139		1955	140	1955

[A] The Model with the four parameters (A, k, n and b) included as random did not converge.

Table 6 Statistics of models including different random effect components and different variance structure in the model.

A = the asymptotic limit of the BW when age *t* approaches to infinity; *b* = integration constant; *k* = ratio of the relative intensity of growth; *n* = shape parameter determining the position of the inflection point (POI) of the curve.

Models	M1		M2		M3		M4		M5		M6	
Variance structure	Constant		Power		Constant plus Power							
Random parameters			A k n				A k		A n		A	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
Parameter												
A	590	2.7	569	2.5	568	2.5	577	2.1	569	2.5	577	2.1
b	-0.914	0.0040	-0.801	0.0040	-0.798	0.0041	-0.846	0.0032	-0.800	0.0040	-0.846	0.0032
k (x1000)	2.087	0.0290	2.692	0.0210	2.713	0.0209	2.437	0.0186	2.693	0.0210	2.437	0.0186
n	-0.895	0.0127	-0.615	0.0075	-0.611	0.0075	-0.707	0.0072	-0.614	0.0075	-0.707	0.0072

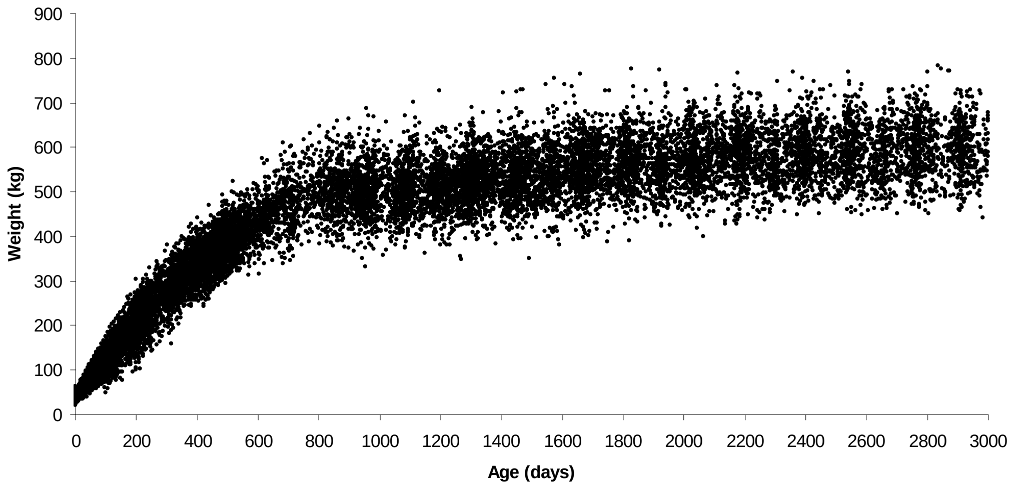


Fig. 1 Distribution of weight records on the data set.

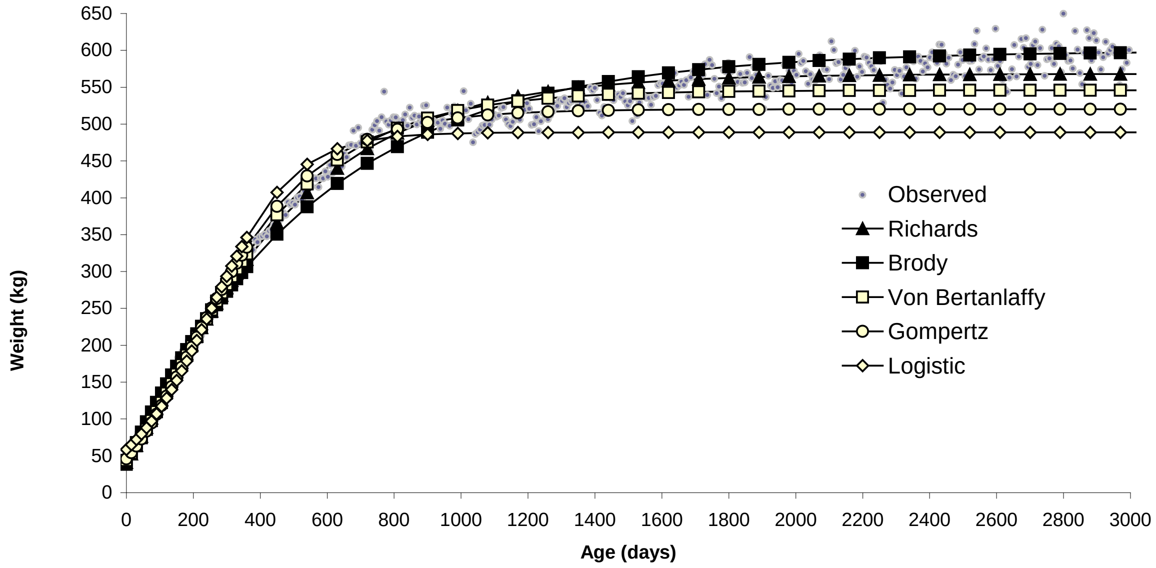
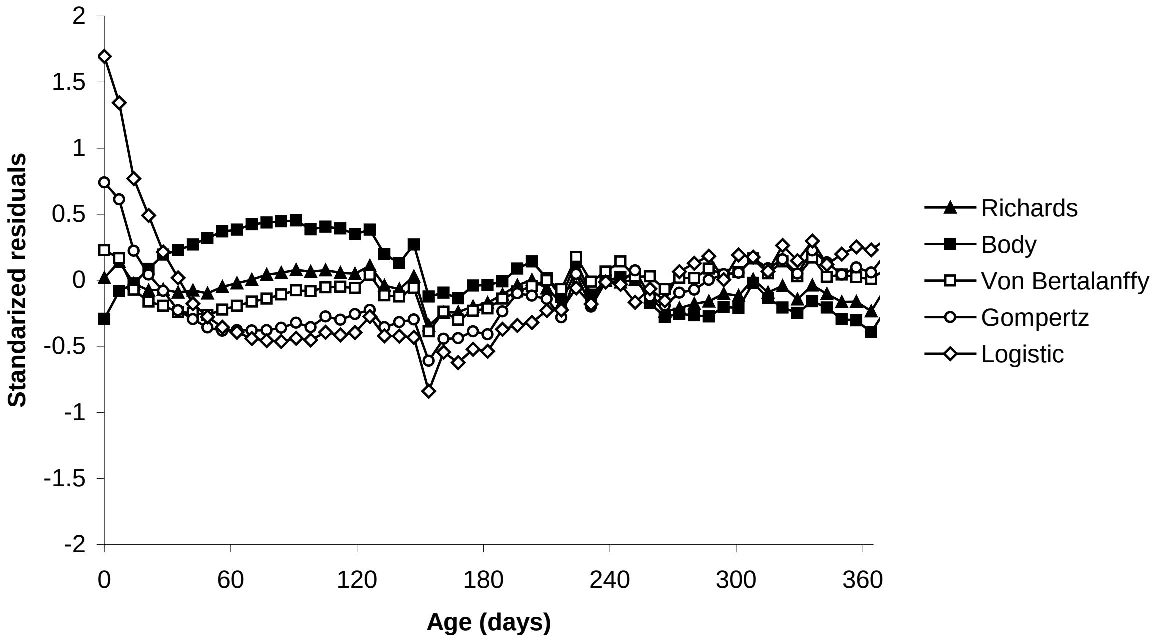


Fig. 2 Weekly average of the observed weights and estimated weights with the different nonlinear functions tested.



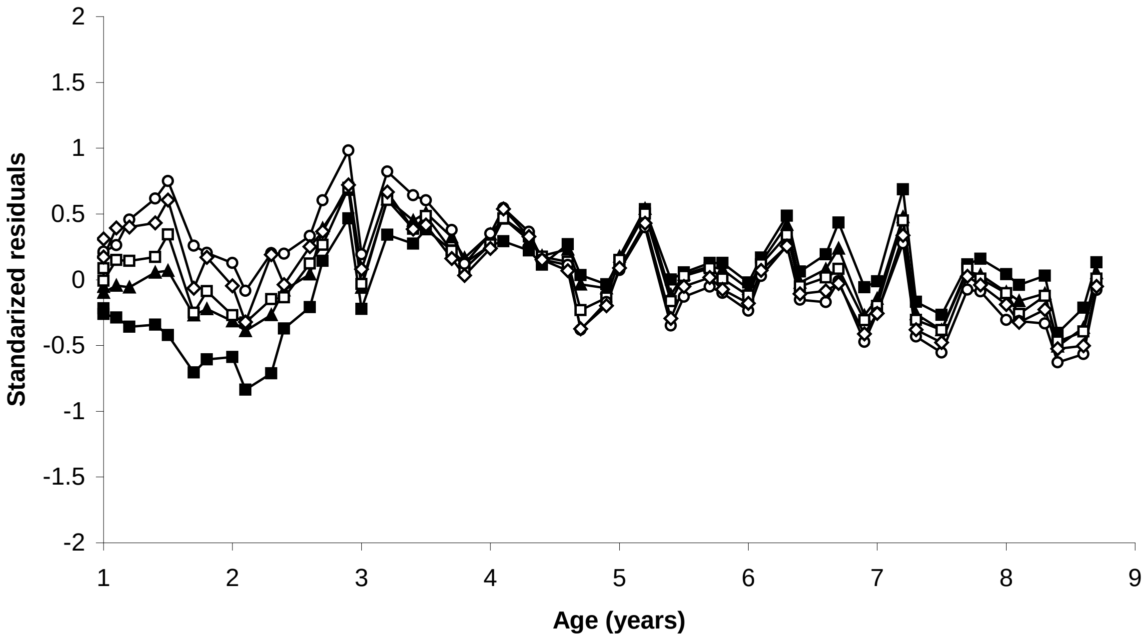


Fig. 3 Standarized residuals of the different nonlinear functions during first year of age (above) and after first year of age age (below).

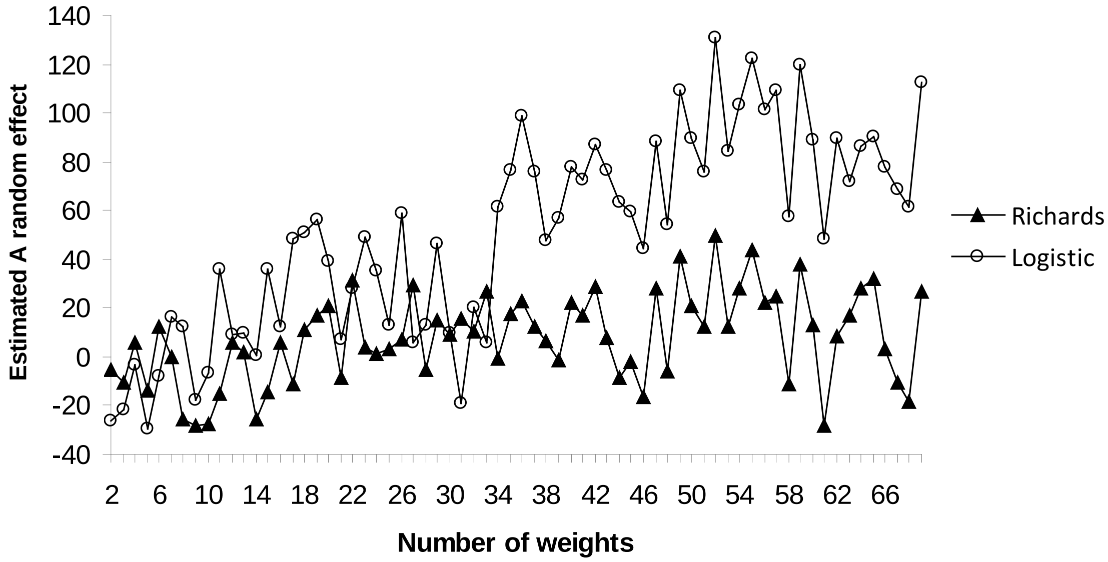


Fig. 4 Estimates of the *A* random effect in the Logistic and Richards functions depending on number of available bodyweights.

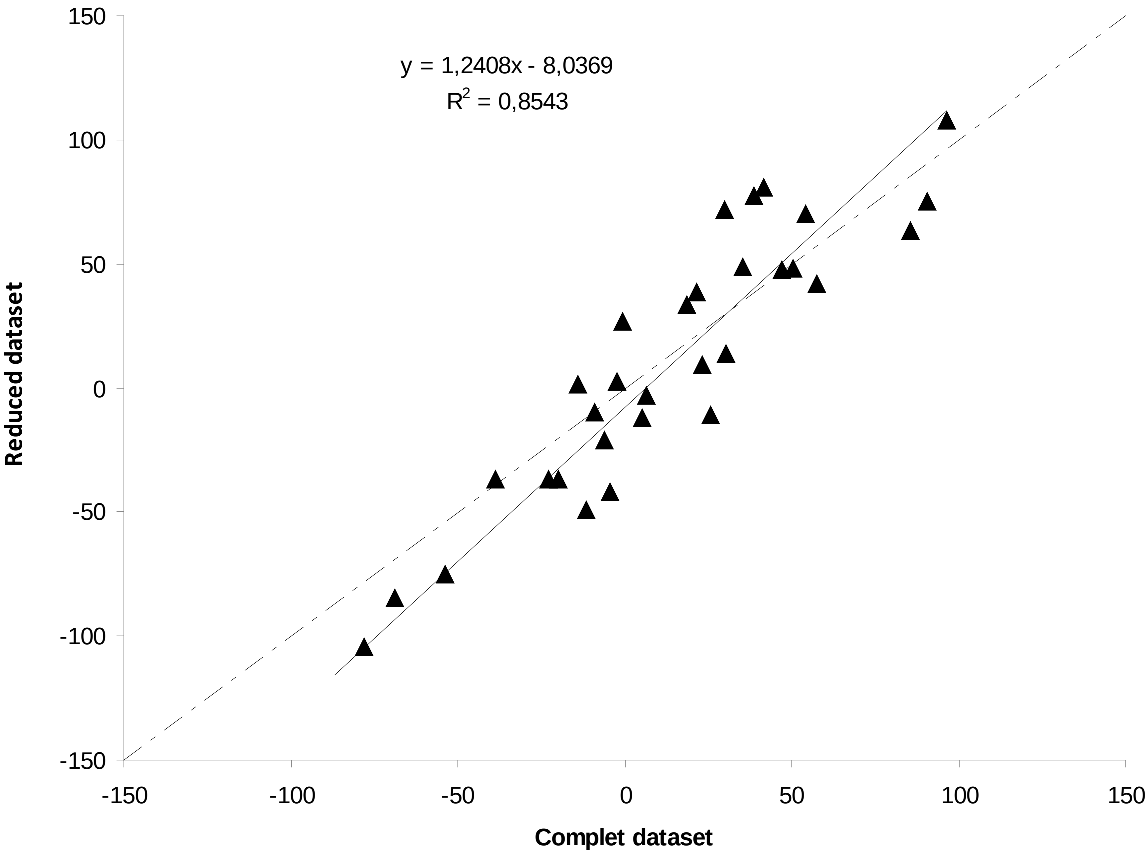


Fig. 5 Estimates of the *A* random effect in the Richards functions obtained with all data available (complete dataset) and using only data until 200 days of age (reduced dataset).